Providing Guaranteed Services Without Per Flow Management

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Abstract

Existing approaches for providing guaranteed services require routers to manage per flow states and perform per flow operations [3, 13]. Such a *stateful* network architecture is less scalable and robust than *stateless* network architectures like the original IP and the recently proposed Diffserv [12]. However, services provided with current *stateless* solutions, Diffserv included, have lower flexibility, utilization, and/or assurance level as compared to the services that can be provided with per flow mechanisms.

In this paper, we propose techniques that do not require per flow management (either control or data planes) at core routers, but can implement guaranteed services with levels of flexibility, utilization, and assurance similar to those that can be provided with per flow mechanisms. In this way we can simultaneously achieve high quality of service, high scalability and robustness. The key technique we use is called Dynamic Packet State (DPS), which provides a lightweight and robust mechanism for routers to coordinate actions and implement distributed algorithms. We present an implementation of the proposed algorithms that has minimum incompatibility with IPv4.

1 Introduction

Current IP networks provide one simple service: the best-effort datagram delivery. Such a simple service model allows IP routers to be stateless: except routing state, which is highly aggregated, routers do not keep any other fine grain information about traffic. Providing a minimalist service model and having the "stateless waist" in the protocol hourglass allows the Internet to scale with both the size of the network and heterogeneous applications and technologies. Together, they are two of the most important technical reasons behind the success of the Internet.

As the Internet evolves into a global communication infrastructure, there is a growing need to support more sophisticated services (e.g., traffic management, QoS) than the traditional best-effort service. Two classes of solutions emerge: those maintaining the *stateless* property of the original IP architecture, and those requiring a new *stateful* architecture. Examples of *stateless* solutions are RED for congestion control [15] and Differentiated Service (Diffserv) [12] for QoS. The corresponding examples of *stateful* solutions are Fair Queueing [11] for congestion control and Integrated Service (Intserv) [3] for QoS. In general, stateful solutions can provide more powerful and flexible services. For example, compared with RED, Fair Queueing can protect well-behaving flows from misbehaving ones and accommodate heterogeneous end-to-end congestion control algorithms [20, 25]. Similarly, as discussed in Section 2, services provided by Intserv solutions have higher flexibility, utilization, and/or assurance level than those provided by Diffserv solutions. However, as also discussed in Section 2, stateful solutions are less scalable and robust than their stateless counterparts.

The question we want to answer is: is it possible to have the best of the two worlds, i.e., providing services as powerful as those implemented by stateful networks, while utilizing algorithms as scalable and robust as those used in stateless networks?

While we cannot answer the above question in its full generality, we can answer it in some specific cases of practical interest. We consider a network architecture similar to the Diffserv architecture, called Scalable Core or SCORE, in which only edge routers perform per flow management, while core routers do not. As illustrated in Figure 1, the goal of a SCORE network is to approximate the service provided by a *reference stateful* network. In [29] we have shown that a SCORE network can achieve fair bandwidth allocation by approximating the service provided by a reference network in which every node performs fair queueing.

In this paper, we will show that a SCORE network can provide end-to-end per flow delay and bandwidth guarantees as defined in Intserv. Current Intserv solutions assume a stateful network in which two types of per flow state are needed: *forwarding state*, which is used by the forwarding engine to ensure fixed path forwarding, and *QoS state*[1], which is used by both the admission

[1] In the context of RSVP, we use "QoS" state to refer to both the flow spec and the filter spec.

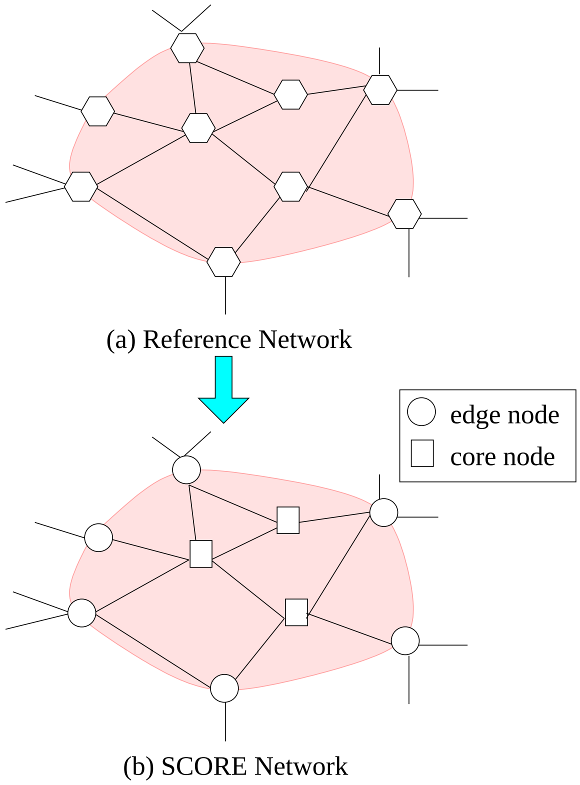


Figure 1: (a) A reference stateful network whose functionality is approximated by (b) a Scalable Core (SCORE) network. In SCORE only edge nodes perform per flow management; core nodes do not perform per flow management.

control module in the control plane and the classifier and scheduler in the data plane. In [30], we have proposed an algorithm that implements fixed path forwarding with no per flow forwarding state. In this paper, we focus on techniques to eliminate the need for core nodes to keep per flow QoS state. In particular, we propose two algorithms: one for the data plane to schedule packets, and the other for the control plane to perform admission control. Neither requires per flow state at core routers.

The key technique used to implement a SCORE network is Dynamic Packet State (DPS). With DPS, each packet carries in its header some state that is initialized by the ingress router. Core routers process each incoming packet based on the state carried in the packet's header, updating both its internal state and the state in the packet's header before forwarding it to the next hop (see Figure 2). By using DPS to coordinate actions of edge and core routers along the path traversed by a flow, distributed algorithms can be designed to approximate the behavior of a broad class of stateful networks using networks in which core routers do not maintain per flow state.

The rest of the paper is organized as follows. In Section 2, we give an overview of Intserv

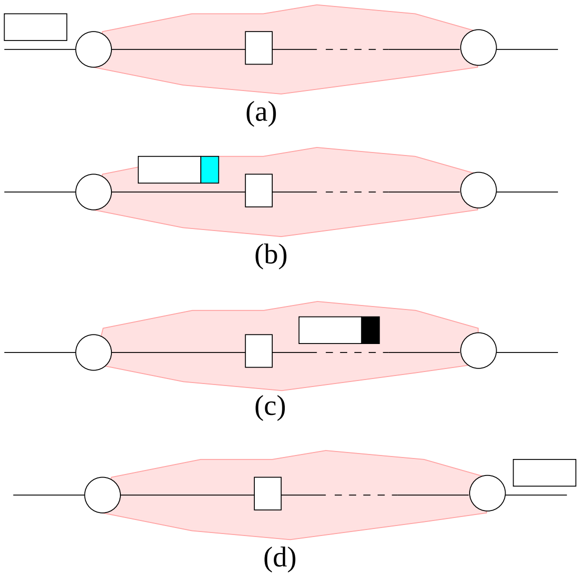


Figure 2: The illustration of the Dynamic Packet State (DPS) technique used to implement a SCORE network: (a-b) upon a packet arrival the ingress node inserts some state into the packet header; (b-c) a core node processes the packet based on this state, and eventually updates both its internal state and the packet state before forwarding it. (c-d) the egress node removes the state from the packet header.

and Diffserv, and discuss the tradeoffs of these two architectures in providing QoS. In Sections 3 and 4 we present the details of our data and control path algorithms, respectively. Section 5 describes a design and a prototype implementation of the proposed algorithms in IPv4 networks. This demonstrates that it is indeed possible to implement algorithms with Dynamic Packet State techniques that have minimum incompatibility with existing protocols. Finally, we conclude the paper in Section 7.

2 Intserv and Diffserv

To support QoS in the Internet, the IETF has defined two architectures: the Integrated Services or Intserv [3], and the Differentiated Services or Diffserv [12]. They have important differences in both service definitions and implementation architectures. At the service definition level, Intserv provides end-to-end guaranteed [26] or controlled load service [37] on a per flow (individual or aggregate) basis, while Diffserv provides a coarser level of service differentiation among a small number of traffic classes. At the implementation level, current Intserv solutions require each router to process *per flow* signaling messages and maintain *per flow* data forwarding and QoS state on the control path, and to perform *per flow* classification, scheduling, and buffer management on the data path. Performing per flow management inside the network affects both the network *scalability* and *robustness*. The former is because the complexities of these per flow operations usually increase as a function of the number of flows; the later is because it is difficult to maintain the consistency of dynamic, and replicated per flow state in a distributed network environment. As pointed out by Clark in [5]: *"because of the distributed nature of the replication, algorithms to ensure robust replication are themselves difficult to build, and few networks with distributed state information provide any sort of protection against failure."* While there are several proposals that aim to reduce the number of flows inside the network by aggregating micro-flows that follow the same path into one macro-flow [2, 18], they only alleviate this problem, but do not fundamentally solve it — the number of macro flows can still be quite large in a network with many edge routers, as the number of paths is a quadratic function of the number of edge nodes.

Diffserv, on the other hand, distinguishes between edge and core routers. While edge routers process packets on the basis of finer traffic granularity, such as per flow or per organization, core routers do not maintain fine grain state, and process packets based on a small number of Per Hop Behaviors (PHBs) encoded by bit patterns in the packet header. By pushing the complexity to the edge and maintaining a simple core, Diffserv's *data plane* is much more scalable than Intserv. However, Diffserv still needs to address the problem of admission control on the control path. One proposal is to use a centralized bandwidth broker that maintains the topology as well as the state of all nodes in the network. In this case, the admission control can be implemented by the broker, eliminating the need for maintaining distributed reservation state. Such a centralized approach is more appropriate for an environment where most flows are long lived, and set-up and tear-down events are rare. To support fine grain and dynamic flows, there may be a need for a distributed broker architecture, in which the broker database is replicated or partitioned. Distributed broker architectures are still an active area of research. One can envision an architecture in which, when a broker receives a request, it makes an acceptance or rejection decision based on its own database, without consulting other brokers. This eliminates the need for a signaling protocol, but requires another protocol to maintain the consistency of the different broker databases. However, since it is impossible to achieve perfect consistency, this may lead to race conditions and/or resource fragmentation. In particular, since requests which arrive simultaneously at different brokers may want to reserve capacity along the same link, each broker can independently allocate only a fraction of the link capacity without running the risk of over-provisioning. This translates into a fundamental trade-off between scalability and fragmentation: while increasing the number of brokers make the solution more scalable, it also increases resource fragmentation.

While Diffserv is more scalable than Intserv in terms of implementation, services provided with existing Diffserv solutions usually have lower flexibility, utilization, and assurance levels than Intserv services. Two examples of differentiated service models are the assured service [6, 7] and the premium service [22]. The assured service is a form of statistical service and achieves lower assurance than guaranteed service. The premium service provides the equivalent of a dedicated link of fixed bandwidth between two edge nodes. However, as we have shown in Appendix A, in order for the premium service to achieve service assurance comparable to the guaranteed service, even with a relative large queueing delay bound (e.g., 200 ms), the fraction of bandwidth that can be allocated to premium service traffic has to be very low (e.g., 10%). It is debatable whether these numbers should be of significant concern. For example, low utilization by the premium traffic may be acceptable if the majority of traffic will be best effort, either because the best effort service is "good enough" for most applications or the price difference between premium traffic and best effort traffic is too high to justify the performance difference between them. Alternatively, if the guaranteed nature of service assurance is not needed, i.e., statistical service assurance is sufficient for premium service, higher network utilization can be achieved. Providing meaningful statistical service is still an open research problem. A discussion of these topics is beyond the scope of this paper. For the remaining sections of the paper, we assume that it is a desirable goal to provide guaranteed service and at the same time achieve high resource utilization.

In summary, Intserv provides more powerful service but has serious limitations with respect to network scalability and robustness. Diffserv is more scalable, but cannot provide services that are comparable to Intserv. In addition, scalable and robust admission control for Diffserv is still an open research problem.

3 QoS Scheduling Without Per Flow State

Current Intserv solutions assume a stateful network in which each router maintains per flow QoS state. The state is used by both the admission control module in the control plane and the classifier and scheduler in the data plane.

In this paper, we propose scheduling and admission control algorithms that provide guarantee services but do not require core routers to maintain per flow state. In this section, we present techniques that eliminate the need for data plane algorithms to use per flow state at core nodes. In particular, at core nodes, packet classification is no longer needed and packet scheduling is based on the state carried in packet headers, rather than per flow state stored locally at each node. In Section 4, we will show that fully distributed admission control can also be achieved without the need for maintaining per flow state at core nodes.

The main idea behind our solution is to approximate a *reference* stateful network with a SCORE network. The key technique used to implement approximation algorithms is Dynamic Packet State (DPS). With DPS, each packet carries some state which is initialized by the ingress node, and then updated by core nodes along the packet's path. The state is used by nodes traversed by the packet to coordinate actions and implement distributed algorithms. On the data path, our algorithm aims to approximate a network with every node implementing the Delay-Jitter-Controlled Virtual Clock (Jitter-VC) algorithm. We make this choice for several reasons. First, unlike various Fair Queueing algorithms [11, 24], in which a packet's deadline can depend on state variables of *all* active flows, in Virtual Clock a packet's deadline depends only on the state variables of the flow it belongs to. This property of Virtual Clock makes the algorithm easier to approximate in a SCORE network. In particular, the fact that the deadline of each packet can be computed exclusively based on the state variables of the flow it belongs to, makes possible to eliminate the need of replicating and maintaining per flow state at all nodes across the path. Instead, per flow state can be stored only at the ingress node, inserted into the packet header by the ingress node, and retrieved later by core nodes, which then use it to determine the packet's deadline. Second, by regulating traffic inside network using delay-jitter-controllers (discussed below), it can be shown that with very high probability, the number of packets in the server at any given time is significantly smaller than the number of flows (see Section 3.3). This helps to simplify the scheduler.

In the remainder of this section, we will first describe the implementation of Jitter-VC using per flow state, then present our algorithm, called Core-Jitter-VC (CJVC), which uses the technique of Dynamic Packet State (DPS). In Appendix B we present an analysis to show that a network of routers implementing CJVC provides the same delay bound as a network of routers implementing the Jitter-VC algorithm.

3.1 Jitter Virtual Clock (Jitter-VC)

Jitter-VC is a non-work-conserving version of the Virtual Clock algorithm [40]. It uses a combination of a delay-jitter rate-controller [33, 39] and a Virtual Clock scheduler. The algorithm works as follows: each packet is assigned an eligible time and a deadline upon its arrival. The packet is held in the rate-controller until it becomes eligible, i.e., the system time exceeds the packet's eligible time (see Figure 3(a)). The scheduler then orders the transmission of eligible packets according to their deadlines.

For the k^{th} packet of flow i, its eligible time $e_{i,j}^k$ and deadline $d_{i,j}^k$ at the j^{th} node on its path are computed as follows:

$$\begin{aligned} e_{i,j}^1 &= a_{i,j}^1 \\ e_{i,j}^k &= \max(a_{i,j}^k + g_{i,j-1}^k, d_{i,j}^{k-1}), \quad i,j \geq 1, k > 1 \qquad (1) \\ d_{i,j}^k &= e_{i,j}^k + \frac{l_i^k}{r_i}, \quad i,j,k \geq 1 \qquad (2) \end{aligned}$$

Notation	Comments
p_i^k	the k-th packet of flow i
l_i^k	length of p_i^k
$a_{i,j}^k$	arrival time of p_i^k at node j
$s_{i,j}^k$	sending time of p_i^k at node j
$e_{i,j}^k$	eligible time of p_i^k at node j
$d_{i,j}^k$	deadline of p_i^k at node j
$g_{i,j}^k$	time ahead of schedule: $g_{i,j}^k = d_{i,j}^k + \tau_j - s_{i,j}^k$
δ_i^k	slack delay of p_i^k
π_j	propagation delay between nodes j and $j+1$
τ_j	transmission time of a maximum size packet at node j

Table 1: Notations used in Section 3.

where l_i^k is the length of the packet, r_i is the reserved rate for the flow, $a_{i,j}^k$ is the packet's arrival time at the j^{th} node traversed by the packet, and $g_{i,j}^k$, stamped into the packet header by the previous node, is the amount of time the packet was transmitted before its schedule, i.e., the difference between the packet's deadline and its actual departure time at the $j-1^{th}$ node. Note that the packet deadline is actually inflated by τ_j, i.e., the transmission time of a packet of maximum size between nodes j and $j+1$. This correction is needed because a packet can miss its deadline by τ_j [40].

Intuitively, the algorithm eliminates the delay variation of different packets by forcing all packets to incur the maximum allowable delay. The purpose of having $g_{i,j-1}^k$ is to compensate at node j the variation of delay due to load fluctuation at the previous node $j-1$. Such regulations limit the traffic burstiness caused by network load fluctuations, and as a consequence, reduce both buffer space requirements and the scheduler complexity.

It has been shown that if a flow's long term arrival rate is no greater than its reserved rate, a network of Virtual Clock servers can provide the same delay guarantee to the flow as a network of WFQ servers [14, 17, 28]. In addition, it has been shown that a network of Jitter-VC servers can provide the same delay guarantees as a network of Virtual Clock servers [10, 16]. Therefore, a network of Jitter-VC servers can provide the same guaranteed service as a network of WFQ servers.

3.2 Core-Jitter-VC (CJVC)

In this section we propose a variant of Jitter-VC, called Core-Jitter-VC (CJVC), which does not require per flow state at core nodes. In addition, we show that a network of CJVC servers can provide the same guaranteed service as a network of Jitter-VC servers.

CJVC uses the DPS technique. The key idea is to have the ingress node to encode scheduling parameters in each packet's header. The core routers can then make scheduling decisions based on the parameters encoded in packet headers, thus eliminating the need for maintaining per flow state at core nodes. As suggested by Eqs. (1) and (2), the Jitter-VC algorithm needs two state variables for each flow i: r_i, which is the reserved rate for flow i and $d_{i,j}^k$, which is the deadline of the last packet from flow i that was served by node j. While it is straightforward to eliminate r_i by putting it in the packet header, it is not trivial to eliminate $d_{i,j}^k$. The difference between r_i and $d_{i,j}^k$ is that while all nodes along the path keep the same r_i value for flow i, $d_{i,j}^k$ is a dynamic value that is computed iteratively at each node. In fact, the eligible time and the deadline of p_i^k depend on the deadline of the previous packet of the same flow, i.e., $d_{i,j}^{k-1}$.

A naive implementation using the DPS technique would be to pre-compute the eligible times and the deadlines of the packet at all nodes along its path and insert all of them in the header. This would eliminate the need for core nodes to maintain $d_{i,j}^k$. The main disadvantage of this approach is that the amount of information carried by the packet increases with the number of hops along the path. The challenge then is to design algorithms that compute $d_{i,j}^k$ for all nodes while requiring a minimum amount of state in the packet header.

Notice that in Eq. (1), the reason for node j to maintain $d_{i,j}^k$ is that it will be used to compute the deadline and the eligible time of the next packet. Since it is only used in a `max` operation, we can eliminate the need for $d_{i,j}^k$ if we can ensure that the other term in `max` is never less than $d_{i,j}^k$. The key idea is then to use a *slack* variable associated with each packet, denoted δ_i^k, such that for every core node j along the path, the following holds

$$a_{i,j}^k + g_{i,j-1}^k + \delta_i^k \geq d_{i,j}^{k-1}, \quad j > 1 \tag{3}$$

By replacing the first term of `max` in Eq. (1) with $a_{i,j}^k + g_{i,j-1}^k + \delta_i^k$, the computation of the eligible time reduces to

$$e_{i,j}^k = a_{i,j}^k + g_{i,j-1}^k + \delta_i^k, \quad j > 1 \tag{4}$$

Therefore, by using one additional DPS variable δ_i^k we eliminate the need for maintaining $d_{i,j}^k$ at the core nodes.

The derivation of δ_i^k proceeds in two steps. First, we express the eligible time of packet p_i^k at an arbitrary core node j, $e_{i,j}^k$, as a function of the eligible time of p_i^k at the ingress node $e_{i,1}^k$ (see Eq. (7)). Second, we use this result and Ineq. (4) to derive a lower bound for δ_i^k.

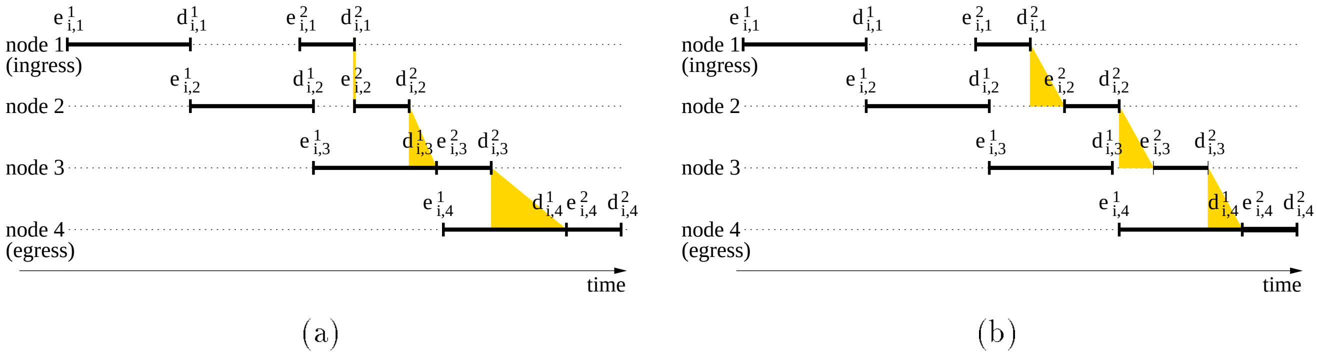


Figure 3: The time diagram of the first two packets of flow i along a four nodes path under (a) Jitter-VC, and (b) CJVC, respectively. Propagation times (π_j) and transmission times of maximum size packets (τ_j) are ignored.

We now proceed with the first step. Recall that $g_{i,j-1}^k$ represents the time by which p_i^k is transmitted before its schedule at node $j-1$, i.e., $d_{i,j-1}^k + \tau_{j-1} - s_{i,j-1}^k$, where τ_{j-1} is the maximum time by which a packet can miss its deadline at node $j-1$. Let π_{j-1} denote the propagation delay between nodes $j-1$ and j. Then the arrival time of p_i^k at node j, $a_{i,j}^k$, is given by

$$\begin{aligned} a_{i,j}^k &= s_{i,j-1}^k + \pi_{j-1} + \tau_{j-1} \\ &= d_{i,j-1}^k - g_{i,j-1}^k + \pi_{j-1} + \tau_{j-1}. \end{aligned} \tag{5}$$

By replacing $a_{i,j}^k$, given by the above expression, in Eq. (4), and then using Eq. (2), we obtain

$$\begin{aligned} e_{i,j}^k &= d_{i,j-1}^k + \delta_i^k + \pi_{j-1} + \tau_{j-1} \\ &= e_{i,j-1}^k + \frac{l_i^k}{r_i} + \delta_i^k + \pi_{j-1} + \tau_{j-1}. \end{aligned} \tag{6}$$

By iterating over the above equation we express $e_{i,j}^k$ as a function of $e_{i,1}^k$:

$$e_{i,j}^k = e_{i,1}^k + (j-1)\left(\frac{l_i^k}{r_i} + \delta_i^k\right) + \sum_{m=1}^{j-1}(\pi_m + \tau_m), \quad j > 1 \tag{7}$$

We are now ready to compute δ_i^k. Recall that the goal is to compute the minimum δ_i^k which ensures that Ineq. (3) holds for every node along the path. After combining Ineq. (3), Eq. (4) and Eq. (2) this reduces to ensure that

$$e_{i,j}^k \geq d_{i,j}^{k-1} \Rightarrow e_{i,j}^k \geq e_{i,j}^{k-1} + \frac{l_i^{k-1}}{r_i}, \quad j > 1 \tag{8}$$

By plugging $e_{i,j}^k$ and $e_{i,j}^{k-1}$ as expressed by Eq. (7) into Ineq. (8), we get

$$\delta_i^k \geq \delta_i^{k-1} + \frac{l_i^{k-1} - l_i^k}{r_i} + \frac{e_{i,1}^{k-1} + l_i^{k-1}/r_i - e_{i,1}^k}{(j-1)}, \quad j > 1 \tag{9}$$

From Eqs. (1) and (2) we have $e_{i,1}^k \geq d_{i,1}^{k-1} = e_{i,1}^{k-1} + l_i^{k-1}/r_i$. Thus, the right-hand side term in Ineq. (9) is maximized when $j = h$. As a result we compute δ_i^k as

$$\begin{aligned} \delta_i^1 &= 0, \\ \delta_i^k &= \max\left(0, \delta_i^{k-1} + \frac{l_i^{k-1} - l_i^k}{r_i} - \frac{e_{i,1}^k - e_{i,1}^{k-1} - l_i^{k-1}/r_i}{h-1}\right), \quad k > 1, h > 1. \end{aligned} \tag{10}$$

In this way, CJVC ensures that the eligible time of every packet p_i^k at node j is no smaller than the deadline of the previous packet of the same flow at node j, i.e., $e_{i,j}^k \geq d_{i,j}^{k-1}$. In addition, the Virtual Clock scheduler ensures that the deadline of every packet is not missed by more than τ_j [40].

In Appendix B, we have shown that a network of CJVC servers provide the same worst case delay bounds as a network of Jitter-VC servers. More precisely, we have proven the following result.

Theorem 1 *The deadline of a packet at the last hop in a network of CJVC servers is equal to the deadline of the same packet in a corresponding network of Jitter-VC servers.*

The example in Figure 3 provides some intuition behind the above result. The basic observation is that, with Jitter-VC, not counting the propagation delay, the difference between the eligible time of packet p_i^k at node j and its deadline at the previous node $j-1$, i.e., $e_{i,j}^k - d_{i,j-1}^k$, never *decreases* as the packet propagates along the path. Consider the second packet in Figure 3. With Jitter-VC, the differences $e_{i,j}^2 - d_{i,j-1}^2$ (represented by the bases of the gray triangles) increase in j. By introducing the slack variable δ_i^k, CJVC *equalizes* these delays. While this change may increase the delay of the packet at intermediate hops, it does not affect the end-to-end delay bound.

Figure 4 shows the computation of the scheduling parameters $e_{i,j}^k$ and $d_{i,j}^k$ by a CJVC server. The number of hops h is computed at the admission time as discussed in Section 4.1.

3.3 Data Path Complexity

While our algorithms do not maintain per flow state at core nodes, there is still the need for core nodes to perform regulation and packet scheduling based on eligible times and deadlines. The

```
ingress node
on packet p arrival
   i = get_flow(p);
   if (first_packet_of_flow(p, i))
      e_i = current_time;
      δ_i = 0;
   else
      δ_i = max(0, δ_i + (l_i − length(p))/r_i −
            max(current_time − d_i, 0)/(h − 1)); /* Eq. (10) */
      e_i = max(current_time, d_i);
   l_i = length(p);
   d_i = e_i + l_i/r_i;
on packet p transmission
   label(p) ← (r_i, d_i − current_time, δ_i);
core/egress node
   on packet p arrival
      (r, g, δ) ← label(p);
      e = current_time + g + δ; /* Eq. (4) */
      d = e + length(p)/r
   on packet p transmission
      if (core node)
         label(p) ← (r, d − current_time, δ);
      else /* this is an egress node */
         clear_label(p);
```

Figure 4: Algorithms performed by ingress, core, and egress nodes at the packet arrival and departure. Note that core and egress nodes do not maintain per flow state.

natural question to ask is: why is this a more scalable scheme than previous solutions requiring per flow management?

There are several scalability bottlenecks for solutions requiring per flow management. On the data path, the expensive operations are per flow classification and scheduling. On the control path, the complexity is the maintenance of consistent and dynamic state in a distributed environment. Among the three, it is easiest to reduce the complexity of the scheduling algorithm as there is a natural tradeoff between the complexity and the flexibility of the scheduler [35]. In fact, a number of techniques have already been proposed to reduce the scheduling complexity, including those requiring constant time complexity [27, 36, 38].

We also note that due to the way we regulate traffic, it can be shown that with very high probability, the number of packets in the server at any given time is significantly smaller than the number of flows. This will further reduce the scheduling complexity and in addition reduce the buffer space requirement. More precisely, in Appendix C we prove the following result.

Theorem 2 *Consider a server traversed by n flows. Assume that the arrival times of the packets from different flows are independent, and that all packets have the same size. Then, for any given probability ε, the queue size at any time instant during a server busy periodic is asymptotically bounded above by s, where*

$$s = \sqrt{\beta n \left(\frac{\ln n}{2} - \frac{\ln \varepsilon}{2} - 1 \right)}, \tag{11}$$

with a probability larger than $1 - \varepsilon$. For identical reservations $\beta = 1$; for heterogeneous reservations $\beta = 3$.

As an example, let $n = 10^6$, and $\varepsilon = 10^{-10}$, which is the same order of magnitude as the probability of a packet being corrupted at the physical layer. Then, by Eq. (11) we obtain $s = 4174$ if all flows have identical reservations, and $s = 7230$ if flows have heterogeneous reservations. Thus the probability of having more packets in the queue than specified by Eq. (11) can be neglected at the level of the entire system even in the context of guaranteed services.

In Table 2 we compare the bounds given by Eq. (11) to simulation results. In each case we report the maximum queue size achieved during the first n time slots of a busy period over 10^5 independent trials. We note that in the case of all flows having identical reservations we are guaranteed that if the queue does not overflow during the first n time slots of a busy period, it will not overflow during the rest of the busy period (see Corollary 1). Since the probability of a buffer to overflow during the first n time slots is no larger than n times the probability of the buffer to overflow during an arbitrary time slot, we use $\varepsilon = \frac{10^{-5}}{n}$ to compute the corresponding

# flows (n)	bound (s)	max. queue size
100	31	28
1,000	109	100
10,000	374	284
100,000	1276	880
1,000,000	4310	2900

(a)

# flows (n)	bound (s)	max. queue size
100	53	30
1,000	188	95
10,000	648	309
100,000	2210	904
1,000,000	7465	2944

(b)

Table 2: The upper bound of the queue size, s, computed by Eq. (11) for $\varepsilon = \frac{10^{-5}}{n}$ (where n is the number of flows) versus the maximum queue size achieved during the first n time slots of a busy period over 10^5 independent trials, during the first n time slots of a busy period: (a) when all flows have identical reservations; (b) when flows' reservations differ by a factor of 20.

bounds.[2]

The results show that our bounds are reasonably close (within a factor of two) when all reservations are identical, but are more conservative when the reservations are different. Finally, we make three comments. First, by performing per packet regulation at every core node, the bounds given by Eq. (11) hold for any core node and are *independent* of the path length. Second, if the flows' arrival patterns are not independent, we can easily enforce this by randomly delaying the first packet from each backlogged period of the flow at ingress nodes. This will increase the end-to-end packet delay by at most the queueing delay of one extra hop. Third, the bounds given by Eq. (11) are asymptotic. In particular, in proving the results in Appendix C we make the assumption that $n \gg s$. However, this a reasonable assumption in practice, as the most interesting cases involve high values for n, and, as suggested by Eq. (11) and the results in Table 2, even for small values of ε (e.g., 10^{-10}), n is much larger than s in these case.

4 Admission Control With No Per Flow State

A key component of any architecture that provides guaranteed services is the admission control. The main job of the admission control is to ensure that the network resources are not over-committed. In particular it has to ensure that the sum of the reservation rates of all flows that traverse any link in the network is no larger than the link capacity, i.e., $\sum_i r_i < C$. A new reservation request is granted if it passes the admission test at each hop along its path. As discussed in Section 2, implementing such a functionality is not trivial: traditional distributed

[2]More formally, let ε' be the probability that the buffer does not overflow during the first n time slots of the busy period. Then by taking $\varepsilon' = n \cdot \varepsilon$, Eq. (11) becomes $s = \sqrt{\beta n(\ln n - (\ln \varepsilon')/2 - 1)}$.

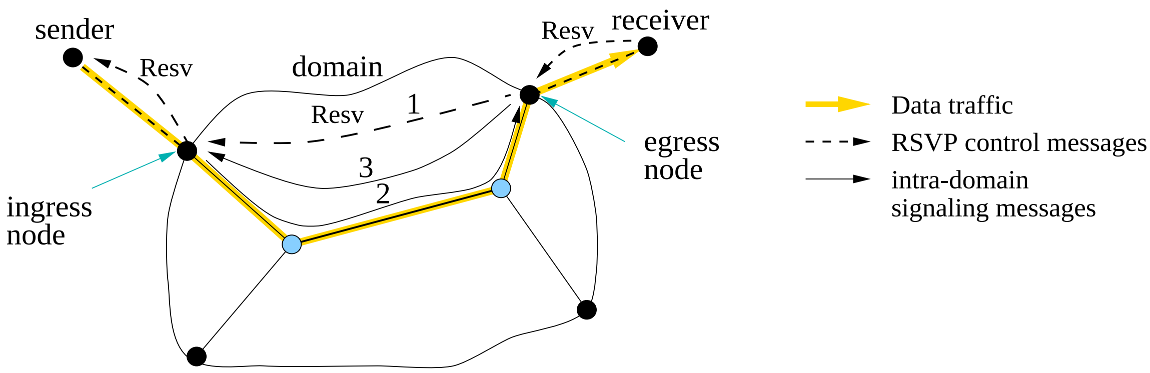


Figure 5: Ingress-egress admission control when RSVP is used outside the SCORE domain.

architectures based on signaling protocols are not scalable and are less robust due to the requirement of maintaining dynamic and replicated state; centralized architectures have scalability and availability concerns.

In this section, we propose a fully distributed architecture for implementing admission control. Like most distributed admission control architectures, in our solution, each node keeps track of the aggregate reservation rate for each of its out-going links and makes local admission control decisions. However, unlike existing reservation protocols, this distributed admission control process is achieved without core nodes maintaining per flow state.

4.1 Ingress-to-Egress Admission Control

We consider an architecture in which a lightweight signaling protocol is used within the SCORE domain. Edge routers are the interface between this signaling protocol and an inter-domain signaling protocol such as RSVP. For the purpose of this discussion, we consider only unicast reservations. In addition, we assume a mechanism like the one proposed in [30] or Multi-Protocol Label Switching (MPLS) [4] that can be used to pin a flow to a route.

From the point of view of RSVP, a path through the SCORE domain is just a virtual link. There are two basic control messages in RSVP: *Path* and *Resv*. These messages are processed only by edge nodes; no operations are performed inside the domain. For the ingress node, upon receiving a *Path* message, it simply forwards it through the domain. For the egress node, upon receiving the first *Resv* message for a flow (i.e., there was no RSVP state for the flow at the egress node before receiving the message), it will forward the message (message "1" in Figure 5) to the corresponding ingress node, which in turn will send a special signaling message (message "2" in Figure 5) along the path toward the egress node. Upon receiving the signaling message, each node along the path performs a local admission control test as described in Section 4.2. In addition, the message carries a counter h that is incremented at each hop. The final value h

is used for computing the slack delay δ (see Eq. (10)). If we use the route pinning mechanism described in [30], message "2" is also used to compute the label of the path between the ingress and egress. This label is used then by the ingress node to make sure that all data packets of the flow are forwarded along the same path. When the signaling message "2" reaches the egress node, it is reflected back to the sender, which makes the final decision (message "3" in Figure 5). RSVP refresh messages for a flow that already has per flow RSVP state installed at edge routers will not trigger additional signaling messages inside the domain.

Since RSVP uses raw IP or UDP to send control messages, there is no need for retransmission for our signaling messages, as message loss will not break the RSVP semantics. If the sender does not receive a reply after a certain timeout, it simply drops the *Resv* message. In addition, as we will show in Section 4.3, there is no need for a special termination message inside the domain when a flow is torn down.

4.2 Per-Hop Admission Control

Each node needs to ensure that $\sum_i r_i < C$ holds at all times. At first sight, one simple solution that implements this test and also avoids per flow state is for each node to maintain the aggregate reserved rate R, where R is updated to $R = R + r$ when a new flow with the reservation rate r is admitted, and to $R = R - r'$ when a flow with the reservation rate r' terminates. The admission control reduces then to checking whether $R + r \leq C$ holds. However, it can be easily shown that such a simple solution is not robust with respect to various failure conditions such as packet loss, partial reservation failures, and network node crashes. To handle packet loss, when a node receives a set-up or tear-down message, the node has to be able to tell whether it is a duplicate of a message already processed. To handle partial reservation failures, a node needs to "remember" what decision it made for the flow in a previous pass. That is why all existing solutions maintain per flow reservation state, be it hard state as in ATM UNI or soft state as in RSVP. However, maintaining *consistent* and *dynamic* state in a *distributed* environment is itself challenging. Fundamentally, this is due to the fact that the update operations assume a *transaction* semantic, which is difficult to implement in a distributed environment [1, 34].

In the remaining of the section, we show that by using DPS, it is possible to significantly reduce the complexity of admission control in a distributed environment. Before we present the details of the algorithm, we point out that our goal is to estimate a close *upper bound* on the aggregate *reserved* rate. By using this bound in the admission test we avoid over-provisioning, which is a necessary condition to provide deterministic service guarantees. This is in contrast to many measurement-based admission control algorithms [19, 32], which, in the context of supporting controlled load or statistical services, base their admission test on the measurement of the *actual* amount of traffic transmitted. To achieve this goal, our algorithm uses two techniques.

Notation	Comments
r_i	flow i's reserved rate
b_i^k	total number of bits flow i is entitled to transmit during $[s_{i,1}^{k-1}, s_{i,1}^k]$, i.e., $b_i^k = r_i(s_{i,1}^k - s_{i,1}^{k-1})$
$R(t)$	aggregate reservation at time t
$R_{bound}(t)$	upper bound of $R(t)$, used by admission test
$R_{DPS}(t)$	estimate of $R(t)$, computed by using DPS
$R_{new}(t)$	sum of all new reservations accepted from the beginning of current estimation interval until t
$R_{cal}(t)$	upper bound of $R(t)$, used to calibrate R_{bound}, computed based on R_{DPS} and R_{new}

Table 3: Notations used in Section 4.3.

First, a conservative upper bound of R, denoted R_{bound}, is maintained at each core node and is used for making admission control decisions. R_{bound} is updated with a simple rule: $R_{bound} = R_{bound} + r$ whenever a new request of a rate r is accepted. It should be noted that in order to maintain the invariant that R_{bound} is an upper bound of R, this algorithm does not need to detect duplicate request messages, generated either due to retransmission in case of packet loss or retry in case of partial reservation failures. Of course, the obvious problem with this algorithm is that R_{bound} will diverge from R. In the limit, when R_{bound} reaches the link capacity C, no new requests can be accepted even though there might be available capacity.

To address this problem, a separate algorithm is introduced to periodically estimate the aggregate reserved rate. Based on this estimate, a second upper bound for R, denoted R_{cal}, is computed and used to re-calibrate R_{bound}. An important aspect of the estimation algorithm is that the discrepancy between the upper bound R_{cal} and the actual reserved rate R can be bounded. The re-calibration then becomes choosing the minimum of the two upper bounds R_{bound} and R_{cal}. The estimation algorithm is based on DPS and does not require core routers to maintain per flow state.

Our algorithms have several important properties. First, they are robust in the presence of network losses and partial reservation failures. Second, while they can over-estimate R, they will never under-estimate R. This ensures the semantics of the guaranteed service – while over-estimation can lead to under-utilization of network resources, under-estimation can result in over-provisioning and violation of performance guarantees. Finally, the proposed estimation algorithms are self-correcting in the sense that over-estimation in a previous period will be corrected in the next period. This greatly reduces the possibility of serious resource under-utilization.

4.3 Aggregate Reservation Estimation Algorithm

In this section, we present the estimation algorithm of the aggregate reserved rate which is performed at each core node. In particular, we will describe how R_{cal} is computed and how it is used to re-calibrate R_{bound}. In designing the algorithm for computing R_{cal}, we want to balance between two goals: (a) R_{cal} should be an upper bound on R; (b) over-estimation errors should be corrected and kept to the minimum.

To compute R_{cal}, we start with an inaccurate estimate of R, denoted R_{DPS}, and then make adjustments to account for estimation inaccuracies. In the following, we first present the algorithm that computes R_{DPS}, then describe the possible inaccuracies and the corresponding adjustment algorithms.

The estimate R_{DPS} is calculated using the DPS technique: ingress nodes insert additional state in packet headers, which is in turn used by core nodes to estimate the aggregate reservation R. In particular, the following state b_i^k is inserted in the header of packet p_i^k:

$$b_i^k = r_i(s_{i,1}^k - s_{i,1}^{k-1}), \tag{12}$$

where $s_{i,1}^{k-1}$ and $s_{i,1}^k$ are the times the packets p_i^{k-1} and p_i^k are transmitted by the ingress node. Therefore, b_i^k represents the total amount of bits that flow i is entitled to send during the interval $[s_{i,1}^{k-1}, s_{i,1}^k]$. The computation of R_{DPS} is based on the following simple observation: the sum of b values of all packets of flow i during an interval is a good approximation for the total number of bits that flow i is *entitled* to send during that interval according to its reserved rate. Similarly, the sum of b values of *all* packets is a good approximation for the total number of bits that all flows are entitled to send during the corresponding interval. Dividing this sum by the length of the interval gives the aggregate reservation rate. More precisely, let us divide time into intervals of length T_W: $(u_k, u_{k+1}]$, $k > 0$. Let $b_i(u_k, u_{k+1})$ be the sum of b values of packets in flow i received during $(u_k, u_{k+1}]$, and let $B(u_k, u_{k+1})$ be the sum of b values of *all* packets during $(u_k, u_{k+1}]$. The estimate is then computed at the end of each interval $(u_k, u_{k+1}]$ as follows

$$R_{DPS}(u_{k+1}) = \frac{B(u_k, u_{k+1})}{u_{k+1} - u_k} = \frac{B(u_k, u_{k+1})}{T_W}. \tag{13}$$

While simple, the above algorithm may introduce two types of inaccuracies. First, it ignores the effects of the delay jitter and the packet inter-departure times. Second, it does not consider the effects of accepting or terminating a reservation in the middle of an estimation interval. In particular, having newly accepted flows in the interval may result in the under-estimation of $R(t)$

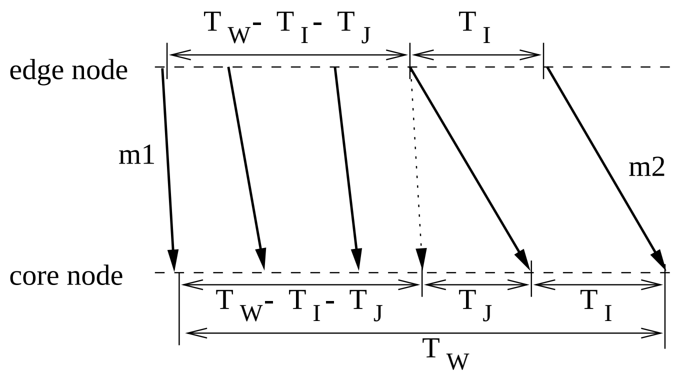


Figure 6: The scenario in which the lower bound of b_i, i.e., $r_i(T_W - T_I - T_J)$, is achieved. The arrows represent packet transmissions. T_W is the averaging window size; T_I is an upper bound on the packet inter-departure time; T_J is an upper bound on the delay jitter. Both $m1$ and $m2$ miss the estimation interval T_W.

by $R_{DPS}(t)$. To illustrate this, consider the following simple example: there are no guaranteed flows on a link until a new request with rate r is accepted at the end of an estimation interval $(u_k, u_{k+1}]$. If no data packet from the new flow reaches the node before u_{k+1}, $B(u_k, u_{k+1})$ would be 0, and so would be $R_{DPS}(u_{k+1})$. However, the correct value should be r.

In the following, we present the algorithm to compute an upper bound of $R(u_{k+1})$, denoted $R_{cal}(u_{k+1})$. In doing this we account for both types of inaccuracies. Let $\mathcal{L}(t)$ denote the set of reservations at time t. Our goal is then to bound the aggregate reservation at time u_{k+1}, i.e., $R(u_{k+1}) = \sum_{i \in \mathcal{L}(u_{k+1})} r_i$. Consider the division of $\mathcal{L}(u_{k+1})$ into two subsets: the subset of new reservations that were accepted during the interval $(u_k, u_{k+1}]$, denoted $\mathcal{N}(u_{k+1})$, and the subset containing the rest of reservations which were accepted no later than u_{k+1}. Next, we express $R(u_{k+1})$ as

$$R(u_{k+1}) = \sum_{i \in \mathcal{L}(u_{k+1}) \setminus \mathcal{N}(u_{k+1})} r_i + \sum_{i \in \mathcal{N}(u_{k+1})} r_i. \tag{14}$$

The idea is then to derive an upper bound for each of the two right-hand side terms, and compute R_{cal} as the sum of these two bounds. To bound $\sum_{i \in \mathcal{L}(u_{k+1}) \setminus \mathcal{N}(u_{k+1})} r_i$, we note that

$$B(u_k, u_{k+1}) \geq \sum_{i \in \mathcal{L}(u_{k+1}) \setminus \mathcal{N}(u_{k+1})} b_i(u_k, u_{k+1}). \tag{15}$$

The reason that (15) is an inequality instead of an equality is that when there are flows terminating during the interval $(u_k, u_{k+1}]$, their packets may still have contributed to $B(u_k, u_{k+1})$ even though they do not belong to $\mathcal{L}(u_{k+1}) \setminus \mathcal{N}(u_{k+1})$. Next, we compute a lower bound for $b_i(u_k, u_{k+1})$. By definition, since $i \in \mathcal{L}(u_{k+1}) \setminus \mathcal{N}(u_{k+1})$, it follows that flow i holds a reservation during the entire interval $(u_k, u_{k+1}]$. Let T_I be the maximum inter-departure time between two consecutive packets of a flow at the edge node, and let T_J be the maximum delay jitter of a flow, where both T_I and T_J are much smaller than T_W. Now, consider the scenario shown in Figure 6 in which a core node receives the packets $m1$ and $m2$ just outside the estimation window. Assuming the worst case in which $m1$ incurs the lowest possible delay, $m2$ incurs the maximum possible delay, and that the last packet before $m2$ departs T_I seconds earlier, it is easy to see that that the sum of the b values carried by the packets received during the estimation interval by the core node cannot be smaller than $r_i(T_W - T_I - T_J)$. Thus, we have

$$\begin{aligned} b_i(u_k, u_{k+1}) &> r_i(T_W - T_I - T_J), && (16) \\ & \quad \forall i \in \mathcal{L}(u_{k+1}) \setminus \mathcal{N}(u_{k+1}). && (17) \end{aligned}$$

By combining Ineqs. (15) and (16), and Eq. (13) we obtain

$$\begin{aligned} \sum_{i \in \mathcal{L}(u_{k+1}) \setminus \mathcal{N}(u_{k+1})} r_i &< \sum_{i \in \mathcal{L}(u_{k+1}) \setminus \mathcal{N}(u_{k+1})} \frac{b_i(u_k, u_{k+1})}{T_W(1-f)} \\ &\leq \frac{R_{DPS}(u_{k+1})}{1-f}, \end{aligned} \tag{18}$$

where $f = (T_I + T_J)/T_W$.

Next, we bound the second right-hand side term in Ineq. (14): $\sum_{i \in \mathcal{N}(u_{k+1})} r_i$ For this, we introduce a new global variable R_{new}. R_{new} is initialized at the beginning of each interval $(u_k, u_{k+1}]$ to zero, and is updated to $R_{new} + r$ every time a new reservation r is accepted. Let $R_{new}(t)$ denote the value of this variable at time t. For simplicity, here we assume that a flow which is granted a reservation during the interval $(u_k, u_{k+1}]$ becomes active no later than u_{k+1}.[3] Then it is easy to see that

$$\sum_{i \in \mathcal{N}(u_{k+1})} r_i \leq R_{new}(u_{k+1}). \tag{19}$$

The inequality holds when no duplicate reservation requests are processed, and none of the new accepted reservations terminate during the interval. Then we define $R_{cal}(u_{k+1})$ as

$$R_{cal}(u_{k+1}) = \frac{R_{DPS}(u_{k+1})}{1-f} + R_{new}(u_{k+1}). \tag{20}$$

From Eq. (14), and Ineqs. (18) and (19) follow easily that $R_{cal}(u_{k+1})$ is an upper bound for $R(u_{k+1})$, i.e., $R_{cal}(u_{k+1}) > R(u_{k+1})$. Finally, we use $R_{cal}(u_{k+1})$ to re-calibrate the upper bound of the aggregate reservation, R_{bound}, at u_{k+1} as

[3] Otherwise, to account for the worst case in which a reservation that was accepted by the node during $(u_{k-1}, u_k]$ becomes at time $u_k + RTT$, we need to subtract $RTT \times R_{new}(u_k)$ from $B(u_k, u_{k+1})$.

Per-hop Admission Control

on reservation request r
if $(R_{bound} + r \leq C)$ /* *perform admission test* */
  $R_{new} = R_{new} + r$;
  $R_{bound} = R_{bound} + r$;
  accept request;
else
  deny request;
on reservation termination r /* *optional* */
  $R_{bound} = R_{bound} - r$;

Aggregate Reservation Bound Comp.

on packet arrival p
  $b \leftarrow get\_b(p)$; /* *get b value inserted by ingress (Eq. (12))* */
  $L = L + b$;
on time-out T_W
  $R_{DPS} = L/T_W$; /* *estimate aggregate reservation* */
  $R_{bound} = \min(R_{bound}, R_{DPS}/(1-f) + R_{new})$;
  $R_{new} = 0$;

Figure 7: The control path algorithms executed by core nodes; R_{new} is initialized to 0.

$$R_{bound}(u_{k+1}) = \min(R_{bound}(u_{k+1}), R_{cal}(u_{k+1})). \tag{21}$$

Figure 7 shows the pseudocode of control algorithms at core nodes. Next we make several observations.

First, the estimation algorithm uses only the information in the current interval. This makes the algorithm robust with respect to loss and duplication of signaling packets since their effects are "forgotten" after one time interval. As an example, if a node processes both the original and a duplicate of the same reservation request during the interval $(u_k, u_{k+1}]$, R_{bound} will be updated twice for the same flow. However, this erroneous update will not be reflected in the computation of $R_{DPS}(u_{k+2})$, since its computation is based only on the b values received during $(u_{k+1}, u_{k+2}]$.

As a consequence, an important property of our admission control algorithm is that it can asymptotically reach a link utilization of $C(1-f)/(1+f)$. In particular, the following result is proven in Appendix D:

Theorem 3 *Consider a link of capacity C at time t. Assume that no reservation terminates and there are no reservation failures or request losses after time t. Then if there is sufficient demand after t the link utilization approaches asymptotically $C(1-f)/(1+f)$.*

Second, note that since $R_{cal}(u_k)$ is an upper bound of $R(u_k)$, a simple solution would be to use $R_{cal}(u_k) + R_{new}$, instead of R_{bound}, to perform the admission test during $(u_k, u_{k+1}]$. The problem with this approach is that R_{cal} can overestimate the aggregate reservation R. An example is given in Section 5.3 to illustrate this issue (Figure 13(b)).

Third, we note that a possible optimization of the admission control algorithm is to add reservation termination messages (see Figure 7). This will reduce the discrepancy between the upper bound R_{bound} and the aggregate reservation R. However, in order to guarantee that R_{bound} remains an upper bound for R, we need to ensure that a termination message is sent at most *once*, i.e., there are no retransmissions if the message is lost. In practice, this property can be enforced by edge nodes, which maintain per flow state.

Finally, to ensure that the maximum inter-departure time is no larger than T_I, the ingress node may need to send a dummy packet in the case when no data packet arrives for a flow during an interval T_I. This can be achieved by having the ingress node to maintain a timer with each flow. An optimization would be to aggregate all "micro-flows" between each pair of ingress and egress nodes into one flow, and compute b values based on the aggregated reservation rate, and insert a dummy packet only if there is no data packet of the aggregate flow during an interval.

5 Implementation and Experiments

The key technique of our algorithms is DPS, which encodes states in the packet header, and thus eliminates the need for maintaining per flow state at each node. Since there is limited space in protocol headers and most header bits have been allocated, the main challenge of implementing these algorithms is to (a) find space in the packet header for storing DPS variables and at the same time remain fully compatible with current standards and protocols; and (b) efficiently encode state variables so that they fit in the available space without introducing too much inaccuracy.

In the remaining of the section, we will first present how we address the above two problems in the context of IPv4 networks, describe a prototype implementation of our algorithms in FreeBSD v2.2.6, and, finally we give results from experiments in local testbed. The main goal of these experiments is to provide a proof of concept of our design.

5.1 Carrying State in Data Packets

Two possibilities to encode state in the packet header are: (1) introduce a new IP option and insert the option at the ingress router, or (2) introduce a new header between layer 2 and layer 3, similar to the way labels are transported in Multi-Protocol Label Switching (MPLS) [4]. While both of these solutions are quite general and can potentially provide large space for encoding state variables, for the propose of our implementation we consider a third option: store the state in the IP header. By doing this, we avoid the penalty imposed by most IPv4 routers in processing the IP options, or the need of devising different solutions for different technologies as it would have been required by introducing a new header between layer 2 and layer 3.

The biggest problem with using the IP header is to find enough space to insert the extra information. The main challenge is to remain compatible with current standards and protocols. In particular, we want the network domain to be transparent to end-to-end protocols, i.e., the egress node should restore the fields changed by ingress and core nodes to their original values. To achieve this goal, we first use four bits from the type of service (TOS) byte (now renamed the Differentiated Service (DS) field) bits which are specifically allocated for local and experimental use [21]. In addition, we observe that there is an *ip_off* field of 13 bits in the IPv4 header to support packet fragmentation/reassembly which is rarely used. For example, by analyzing the traces of over 1.7 million packets on an OC-3 link [23], we found that less than 0.22% of all packets were fragments. Therefore, in most cases it is possible to use *ip_off* field to encode the DPS values. This idea can be implemented as follows. When a packet arrives at an ingress node, the node checks whether a packet is a fragment or needs to be fragmented. If neither of these are true, the *ip_off* field in the packet header will be used to encode DPS values. When the packet reaches the egress node, the *ip_off* is cleared. Otherwise, if the packet is a fragment, it is forwarded as a best-effort packet. In this way the use of *ip_off* is transparent outside the domain. We believe that forwarding a fragment as a best-effort packet, is acceptable in practice, as end-points can easily avoid fragmentation by using an MTU discovery mechanism. Also note that in the above we implicitly assume that packets can be fragmented only by egress nodes.

In summary, we have up to 17 bits available in the current IPv4 header to encode four state variables. The next section discusses how we use this space to encode the DPS states.

5.2 State Encoding

There are four pieces of state that need to be encoded: three are for scheduling purposes, (1) the reserved rate r or equivalently l/r, (2) the slack delay δ, as computed by Eq. (10), and (3) the amount of time g by which the packet was transmitted ahead of schedule at the previous node; and one for admission control purpose, (4) b, as computed by Eq. (12). All are positive values.

```
void intToFP(int val, int *mantissa, int *exponent) {
  int nbits = get_num_bits(val);
  if (nbits <= m) {
    *mantissa = val;
    *exponent = (1 << n) - 1;
  } else {
    *exponent = nbits - m - 1;
    *mantissa = (val >> *exponent) - (1 << m);
  }
}

int FPToInt(int mantissa, int exponent) {
  int tmp;
  if (exponent == ((1 << n) - 1))
    return mantissa;
  tmp = mantissa | (1 << m);
  return (tmp << exponent)
}
```

Figure 8: The C code for converting between integer and floating point formats. m represents the number of bits used by the mantissa; n represents the number of bits in the exponent. Only positive values are represented. The exponent is computed such that the first bit of the mantissa is always 1, when the number is $\geq 2^m$. By omitting this bit, we gain an extra bit in precision. If the number is $< 2^m$ we set by convention the exponent to $2^n - 1$ to indicate this.

One possible solution is to restrict each state variable to only a small number of possible values. For example if a state variable is limited to eight values, only three bits are needed to represent it. While this can be a reasonable solution in practice, in our implementation we use a more sophisticated representation. Basically, we use a floating point like format to represent the *largest* value, and then represent the other value(s) as a fraction of the largest value. In this way we are able to represents a much larger range of possible values. Since computing the eligible time and the deadline involves only additions over these values, our representation achieves good accuracy in terms of relative error. To further optimize the use of the available space we employ two additional techniques. First, we use the floating point format only to represent the *largest* value, and then represent the other value(s) as a fraction of the largest value. In this way we are able to represents a much larger range of possible values. Second, in the case in which there are states which are not required to be simultaneously encoded in the same packet, we use the same

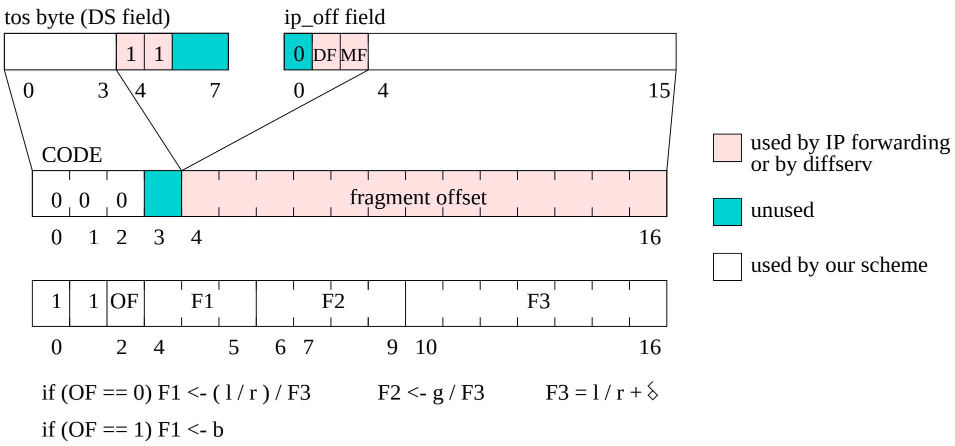


Figure 9: For carrying state we use the four bits from the TOS byte (or DS field) reserved for local use and experimental purposes, and up to 13 bits from the *ip_off*. The first three bits specify whether *ip_off* is used to encode DPS variables. F1, F2, and F3 are used to encode the DPS variables corresponding to a data packet (codes 11x identify the state in data packet headers).

field to encode them. Next, we present the floating point like format.

Assume that a is the largest value carried by the packet, where a is a positive integer. To represent a we use an m bit mantissa and an n bit exponent. Since $a \geq 0$, it is possible to gain an extra bit for mantissa. For this we consider two cases: (a) if $a \geq 2^m$ we represent a as the closest value of the form $u2^v$, where $2^m \leq u \leq 2^{m+1}$. Then, since the $m+1$-th most significant bit in the u's representation is always 1, we can ignore it. As an example, assume $m = 3$, $n = 4$, and $a = 19 = 10011$. Then 19 is represented as $18 = u \times 2^v$, where $u = 9 = 1001$ and $v = 1$. By ignoring the first bit in the representation of u the mantissa will store 001, while the exponent will be 1. (b) On the other hand, if $a < 2^m$, the mantissa will contain a, while the exponent will be $2^n - 1$. For example, for $m = 3$, $n = 4$, and $a = 6 = 110$, the mantissa is 110, while the exponent is 1111. Converting from one format to another can be efficiently implemented. Figure 8 shows the conversion code in C. For simplicity, we assume that integers are truncated rather than rounded when represented in floating point.

By using m bits for mantissa and n for exponent, we can represent any integer in the range $[0..(2^{m+1}-1)\times(2^{2^n-1})]$ with a relative error bounded by $(-1/2^{m+1}, 1/2^{m+1})$. For example, with 7 bits, by allocating 3 for mantissa and 4 for exponent, we can represent any integer in the range $[1..15 \times 2^{15}]$ with a relative error of $(-6.25\%, 6.25\%)$.[4]

If another value $b \leq a$ is carried by the packet we store it as the fraction $f = b/a$. Assuming

[4]The worst relative error case occurs when the mantissa is 8. For example the number $a = 271 = 100001111$ is encoded as $u = 1000$, $v = 5$, with a relative error of $(8 \times 2^5 - 271)/271 = -0.0554 = -5.54\%$. Similarly, $a = 273 = 100010001$ is encoded as $u = 1001$, $v = 5$, with a relative error of 5.55%.

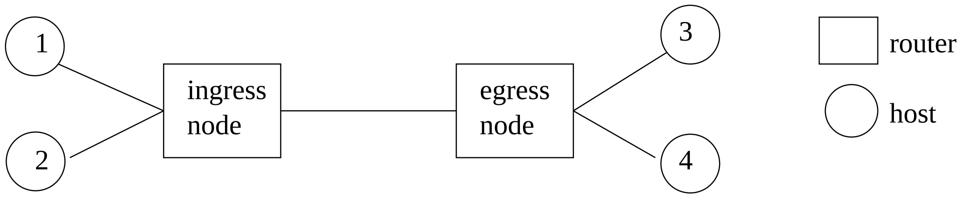


Figure 10: The test configuration used in experiments.

that we use m_1 bits to represent f, the *absolute* error is bounded by $(-1/(2(2^{m_1}-1)), 1/(2(2^{m_1}-1)))$. The -1 in the denominators is a result of mapping 2^{m_1} values to [0, 1], with $2^{m_1}-1$ representing 1. Finally, it is easy to show that by representing a in floating point format with m bits for mantissa and n bits for exponent, and by using m_1 bits to encode b, the *relative* error of $a+b$, denoted $RelErr(a+b)$, is bounded by

$$-\frac{1}{2^{m+1}} - \frac{1}{2^{m_1+1}-2} < RelErr(a+b) < \frac{1}{2^{m+1}} + \frac{1}{2^{m_1+1}-2}, \tag{22}$$

where we ignore the second order term $1/(2^{m+1}(2^{m_1+1}-2))$.

Figure 9 shows how the 17 bits available in the current IPv4 header are used to encode DPS states in a data packet. The 17 bits are divided in four fields: a *code* field which specifies whether the *ip_off* is used to encode state variables, and three *data* fields, denoted $F1$, $F2$ and $F3$, used to encode our variables.

The code field consists of three bits: 000 means that the packet is a fragment and therefore no state is encoded; any other value means that up to 13 bits of *ip_off* are used to encode the state. In particular, the code values specify the layout and the states encoded in the packet header. For example, 11x specifies that the encoded states correspond to a data packet, while 100 specifies that the encoded states correspond to a dummy packet. Due to space limitations, in Figure 9 we show the state encoding for a data packet only. In this case, the last bit of the code field, also called *Offset Field* (*OF*), determines the content of $F1$. If this bit is 1, then $F1$ encodes the b value. Otherwise it encodes $(l/r)/F3$, where $F3 = l/r + \delta$. Finally, $F2$ encodes $g/F3$. We make several observations. First, since $F3$ encodes the largest value among all fields, we represent it in floating point format. By using this format, with seven bits we can represent any positive number in the range $[1..15 \times 2^{15}]$, with a relative error within $(-6.25\%, 6.25\%)$. Second, since the deadline determines the delay guarantees, we use a representation that trades the eligible time accuracy[5] for the deadline accuracy. In particular, the deadline is computed as $d = current\_time + F2 * F3 + F3 \simeq current\_time + g + l/r + \delta$. If *OF* is 0, the eligible time is computed as $e = d - F1 * F3 \simeq current\_time + g + \delta$. $F1$ uses only three bits and its value is

[5]As long as the eligible time value is under-estimated, its inaccuracy will affect only the scheduling complexity, as the packet may become eligible earlier.

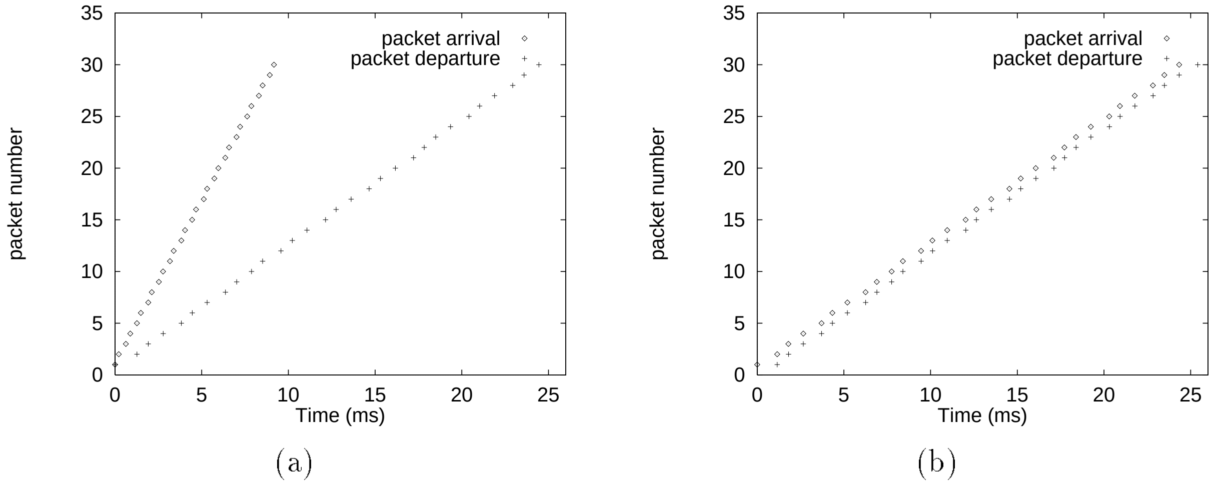


Figure 11: Packet arrival and departure times for a 10 Mbps flow at (a) the ingress node, and (b) the egress node.

computed such that $F1 * F3$ always over-estimates l/r. If OF is 1, the eligible time is computed simply as $e = current\_time$. Third, we express b in units equals with the maximum packet size. In this way we eliminate the need for each packet to carry the b value. In fact, if a flow sends at its reserved rate, only one packet every other eight packets needs to carry the b value. This observation, combined with the fact that the under-estimation of the packet eligible time does not affect the guaranteed delay of the flow, allows us to alternatively encode either b or $(l/r)/F3$ in $F1$, without impacting the correctness of our algorithms.

5.3 Experimental Results

We have implemented these algorithms in FreeBSD v2.2.6 and deployed them in a testbed consisting of 266 MHz and 300 MHz Pentium II PCs connected by point-to-point 100 Mbps Ethernets. The testbed allows configuring a path with up to two intermediate routers.

In the following, we present results from four simple experiments. The experiments are designed to illustrate the microscopic behaviors of the algorithms, rather than their scalability. All experiments were run on the topology shown in Figure 10. The first router is configured as an ingress node, while the second router is configured as an egress node. An egress node also implements the functionalities of a core node. In addition, it restores the initial values of the *ip_off* field. All traffic is UDP and all packets are 1000 bytes, not including the header.

In the first experiment we consider a flow between hosts 1 and 3 that has a reservation of 10 Mbps but sends at a much higher rate of about 30Mbps. Figures 11(a) and (b) plot the arrival and departure times for the first 30 packets of the flow at the ingress and egress node, respectively. One thing to notice in Figure 11(a) is that the arrival rate at the ingress node

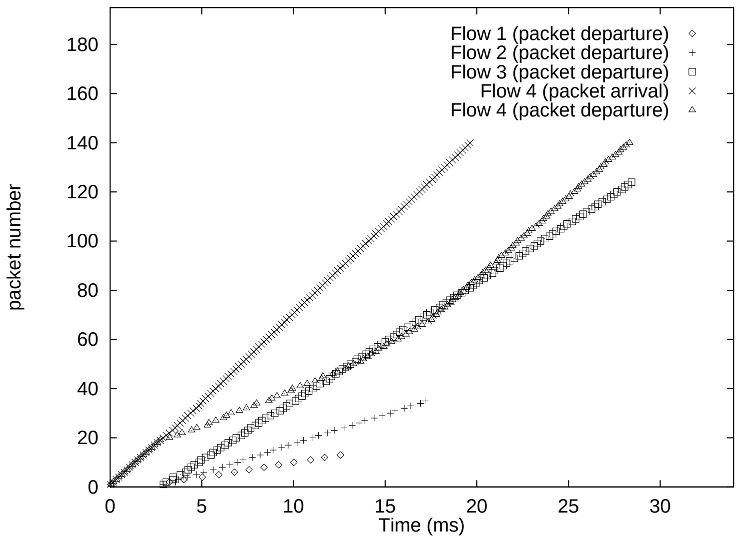


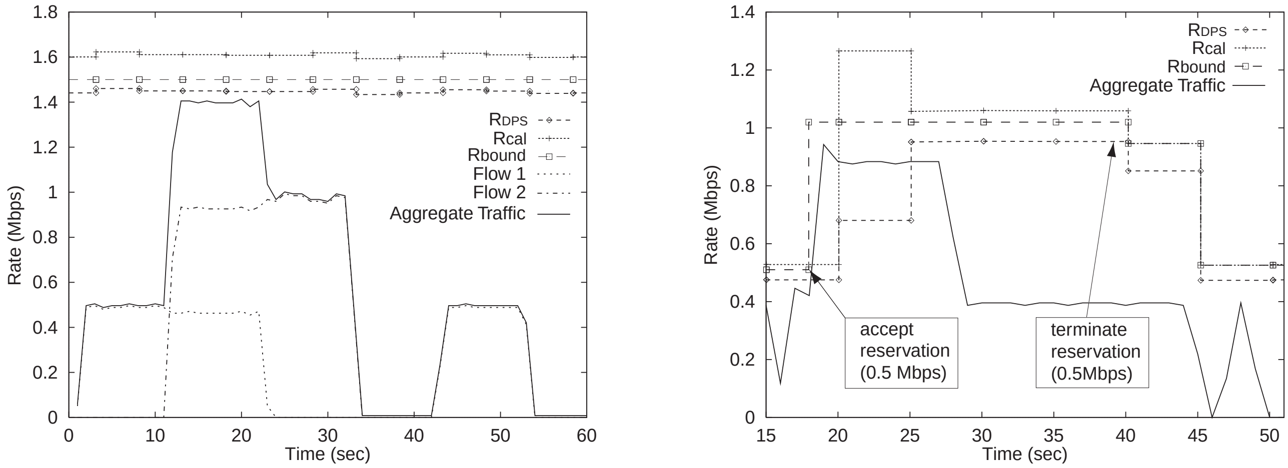
Figure 12: The packets' arrival and departure times for four flows. The first three flows are guaranteed, with reservations of 10 Mbps, 20 Mbps, and 40 Mbps. The last flow is best effort with an arrival rate of about 60 Mbps.

is almost three times the departure rate, which is the same as the reserved rate of 10 Mbps. This illustrate the non-work-conserving nature of the CJVC algorithm, which enforces the traffic profile and allows only 10 Mbps traffic into the network. Another thing to notice is that all packets incur about 0.8 ms delay in the egress node. This is because they are sent by the ingress node as soon as they become eligible, and therefore $g \simeq l/r = 8 \times 1052\text{bits}/10\text{Mbps} = 0.84$ ms. As a result, they will be held in the rate-controller for this amount of time at the next hop[6], which is the egress node in our case.

c

In the second experiment we consider three guaranteed flows between hosts 1 and 3 with reservations of 10 Mbps, 20 Mbps, and 40 Mbps, respectively. In addition, we consider a fourth UDP flow between hosts 2 and 4 which is treated as best effort. The arrival rates of the first three flows are slightly larger than their reservations, while the arrival rate of the fourth flow is approximately 60 Mbps. At time 0, only the best-effort flow is active. At time 2.8 ms, the first three flows become simultaneously active. Flows 1 and 2 terminate after sending 12 and 35 packets, respectively. Figure 12 shows the packet arrival and departure times for the best-effort flow 4, and the packet departure times for the real-time flows 1, 2, and 3. As can be seen, the best-effort packets experience very low delay in the initial period of 2.8 ms. After the QoS flows become active, best-effort packets experience longer delays while QoS flows receive service at their reserved rate. After flow 1 and 2 terminate, the best-effort traffic grabs the remaining

[6]Note that since all packets have the same size, $\delta = 0$.



(a) (b)

Figure 13: The estimate aggregate reservation R_{cal}, and the bounds R_{bound} and R_{cal} in the case of (a) two ON-OFF flows with reservations of 0.5 Mbps, and 1.5 Mbps, respectively, and in the case when (b) one reservation of 0.5 Mbps is accepted at time $t = 18$ seconds, and then is terminated at $t = 39$ seconds.

bandwidth.

The last two experiments illustrate the algorithms for admission control described in Section 4.3. The first experiment demonstrates the accuracy of estimating the aggregate reservation based on the b values carried in the packet headers. The second experiment illustrates the computation of the aggregate reservation bound, R_{bound}, when a new reservation is accepted or a reservation terminates. In these experiments we use an averaging interval, T_W, of 5 seconds, and a maximum inter-departure time, T_I, of 500 ms. For simplicity, we neglect the delay jitter, i.e., we assume $T_J = 0$. This gives us $f = (T_I + T_J)/T_W = 0.1$.

In the first experiment we consider two flows, one with a reservation of 0.5 Mbps, and the other with a reservation of 1.5 Mbps. Figure 13(a) plots the arrival rate of each flow, as well as the arrival rate of the aggregate traffic. In addition, Figure 13(a) plots the bound of the aggregate reservation used by admission test, R_{bound}, the estimate of the aggregate reservation R_{DPS}, and the bound R_{cal} used to recalibrate R_{bound}. According to the pseudocode in Figure 7, both R_{DPS} and R_{cal} are updated at the end of each estimation interval. More precisely, every 5 seconds R_{DPS} is computed based on the b values carried in the packet headers, while R_{cal} is computed as $R_{DPS}/(1-f)+R_{new}$. Note that since in this case no new reservation is accepted, we have $R_{new} = 0$, which yields $R_{cal} = R_{DPS}/(1 - f)$. The important thing to note in Figure 13(a) is that the rate variation of the actual traffic (represented by the continuous line) has little effect on the accuracy of computing the aggregate reservation estimate R_{DPS}, and consequently of R_{cal}. In contrast, traditional measurement based admission control algorithms, which base their

	Baseline		1 flow				10 flows				100 flows			
			ingress		egress		ingress		egress		ingress		egress	
	avg	std	avg	std	avg	std	avg	std	avg	std	avg	std	avg	std
enqueue	1.03	0.91	5.02	1.63	4.38	1.55	5.36	1.75	4.60	1.60	5.91	1.81	5.40	2.33
dequeue	1.52	1.91	3.14	3.27	2.69	2.81	2.79	3.68	2.30	2.91	2.77	2.82	1.73	2.12

Table 4: The average and standard deviation of the enqueue and dequeue times, measured in μs.

estimation on the *actual* traffic, would significantly under-estimate the aggregate reservation, especially during the time periods when no data packets are received. In addition, note that since in this experiment R_{cal} is always larger than R_{bound}, and no new reservations are accepted, the value of R_{bound} is never updated.

In the second experiment we consider a scenario in which a new reservation of 0.5 Mbps is accepted at time $t = 18$ seconds and terminates approximately at time $t = 39$ seconds. For the entire time duration, plotted in Figure 13(b), we have a background traffic with an aggregate reservation of 0.5 Mbps. Similarly to the previous case, we plot the rate of the aggregate traffic, and, in addition, R_{bound}, R_{cal}, and R_{DPS}. There are several points worth noting. First, when the reservation is accepted at time $t = 18$ seconds, R_{bound} increases by the value of the accepted reservation, i.e., 0.5 Mbps (see Figure 7). In this way, R_{bound} is guaranteed to remain an upper bound of the aggregate reservation R. In contrast, since both R_{DPS} and R_{cal} are updated only at the end of the estimation interval, they under-estimate the aggregate reservation, as well as the aggregate traffic, before time $t = 20$ seconds. Second, after R_{cal} is updated at time $t = 20$ seconds, as $R_{DPS}/(1 - f) + R_{new}$, the new value significantly over-estimates the aggregate reservation. This is the main reason for which we do not use R_{cal} $(+R_{new})$, but R_{bound}, to do the admission control test. Third, note that unlike the case when the reservation was accepted, R_{bound} does not change when the reservation terminates at time $t = 39$ seconds. This is simply because in our implementation no tear-down message is generated when a reservation terminates. However, as R_{cal} is updated at the end of the next estimation interval (i.e., at time $t = 45$ seconds), R_{bound} drops to the correct value of 0.5 Mbps. This shows the importance of using R_{cal} to recalibrate R_{bound}. In addition, this illustrates the robustness of our algorithm, i.e., the over-estimation in a previous period is corrected in the next period. Finally, note that in both experiments R_{DPS} always under-estimates the aggregate reservation. This is due to the truncation errors in computing both the b values and the R_{DPS} estimate.

5.4 Processing Overhead

To evaluate the overhead of our algorithm we have performed three experiments on a 300 MHz Pentium II involving 1, 10, and 100 flows, respectively. The reservation and actual sending rates of all flows are identical. The aggregate sending rate is about 20% larger than the aggregate reservation rate. Table 4 shows the means and the standard deviations for the enqueue and dequeue times at both ingress and egress nodes. Each of these numbers is based on a measurement of 1000 packets. For comparison we also show the enqueue and dequeue times for the unmodified code. There are several points worth noting. First, our implementation adds less than 5 μs overhead per enqueue operation, and about 2 μs per dequeue operation. In addition, both the enqueue and dequeue times at the ingress node are greater than at the egress node. This is because ingress node performs per flow operations. Furthermore, as the number of flows increases the enqueue times increase only slightly, i.e., by less than 20%. This suggests that our algorithm is indeed scalable in the number of flows. Finally, the dequeue times actually *decrease* as the number of flows increases. This is because the rate-controller is implemented as a calendar queue with each entry corresponding to a 128 μs time interval. Packets with eligible times falling between the same interval are stored in the same entry. Therefore, when the number of flows is large, more packets are stored in the same calendar queue entry. Since all these packets are transferred during one operation when they become eligible, the actual overhead per packet decreases.

6 Related Work

Our scheme shares its intellectual roots with two pieces of related work: Diffserv and the Core-Stateless Fair Queueing.

The idea of implementing QoS services by using a core-stateless architecture was first proposed by Jacobson [22] and Clark [7], and is now being pursued by the IETF Diffserv working group [12]. There are several differences between our scheme and the existing Diffserv proposals. First, our algorithms operate at a much finer granularity both in terms of time and traffic aggregates: the state embedded in a packet can be highly dynamic, as it encodes the *current* state of the flow, rather than the static and global properties such as dropping or scheduling priority. In addition, the goal of our scheme is to implement distributed algorithms that try to approximate the services provided by a network in which all routers implement per flow management. Therefore, we can provide service differentiation and performance guarantees on a *per flow* basis. In contrast, existing Diffserv solutions provide service differentiation only among a small number of traffic classes. Finally, we propose fully distributed and dynamic algorithms for implementing both data and control functionalities, where existing Diffserv solutions rely on more centralized and static algorithms for implementing admission control.

We first proposed the idea of using Dynamic Packet State to encode dynamic per flow state in the context of approximating the Fair Queueing algorithm in a SCORE architecture [29]. While algorithms proposed in this paper share the same architecture as CSFQ, there are important differences both in high level goals and low level mechanisms. First, while CSFQ was designed to support best-effort traffic, algorithms proposed here are designed to support guaranteed services. As a consequence, while CSFQ can use a probabilistic forwarding algorithm to statistically approximate the Fair Queueing service, CJVC needs to use more elaborate mechanisms to provide performance guarantees *identical* to those provided by Virtual Clock or Weighted Fair Queueing algorithms. In particular, CJVC uses three types of Dynamic Packet State for scheduling purpose and regulates traffic at each hop. One more type of Dynamic Packet State was used to implement the admission control, which was not needed in CSFQ. Finally, we have proposed a detailed design for encoding the DPS variables in IPv4.

In this paper, we propose a technique to estimate the aggregate reservation rate and use that estimate to perform admission control. While this may look similar to measurement-based admission control algorithms [19, 32], the objectives and thus the techniques are quite different. The measurement-based admission control algorithms are designed to support controlled-load type of services, the estimation is based on the *actual* amount of traffic transmitted in the past, and is usually an *optimistic* estimate in the sense that the estimated aggregate rate is smaller than the aggregate reserved rate. While this has the benefit of increasing the network utilization by the controlled-load service traffic, it has the risk of incurring transient overloads that may cause the degradation of QoS. In contrast, our algorithm aims to support guaranteed service, and the goal is to estimate a close upper bound on the aggregate *reserved* rate even when the the actual arrival rate may vary.

In [9], Cruz proposed a novel scheduling algorithm called SCED+ in the context of ATM networks. In SCED+, virtual circuits sharing a same path segment are aggregated into a virtual path. At each switch, only per virtual path state instead of per virtual circuit needs to be maintained for scheduling purpose. In addition, an algorithm is proposed to compute, the eligible times and the deadlines of a packet at subsequent nodes, when the packet enters a virtual path. We note that by doing this and using DPS to carry this information in the packets' headers, it is possible to remove per path scheduling state from core nodes. However, unlike our solution, SCED+ do not provide per flow delay differentiation within an aggregate. In addition, the SCED+ work focuses on the data path mechanism, while we addresses both data path and control path issues.

7 Conclusion

In this paper, we developed two distributed algorithms that implement QoS scheduling and admission control in a SCORE network where core routers do not maintain per flow state. Combined, these two algorithms significantly enhance the scalability of both the data and control plane mechanisms for implementing guaranteed services, and at the same time, provide guaranteed services with flexibility, utilization, and assurance levels similar to those that can be provided with per flow mechanisms. The key technique used in both algorithms is called Dynamic Packet State (DPS), which provides a lightweight and robust means for routers to coordinate actions and implement distributed algorithms. By presenting a design and prototype implementation of the proposed algorithms in IPv4 networks, we demonstrate that it is indeed possible to apply DPS techniques and have minimum incompatibility with existing protocols.

As a final note, we believe DPS is a powerful concept. By using DPS to coordinate actions of edge and core routers along the path traversed by a flow, distributed algorithms can be designed to approximate the behavior of a broad class of "stateful" networks with networks in which core routers do not maintain per flow state. We observe that it is possible to extend the current Diffserv framework to accommodate algorithms using Dynamic Packet State such as the ones proposed in this paper and Core-Stateless Fair Queueing [29]. The key extension needed is to associate with each Per Hop Behavior (PHB) additional space in the packet header for storing PHB specific Dynamic Packet State [31]. Such a paradigm will significantly increase the flexibility and capabilities of the services that can be built with a Diffserv-like architecture.

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Appendix A: Network Utilization of Premium Service in Diffserv Networks

Premium service provides the equivalent of a dedicated link of fixed bandwidth between edge nodes in a Diffserv network. In such a service, each premium flow has a reserved peak rate. In the data plane, ingress nodes police each premium service traffic flow according to its peak reservation rate. Inside the Diffserv domain, core routers put the aggregate of all premium traffic into one scheduling queue and service the premium traffic with strict priority over best effort traffic. In the control plane, a bandwidth broker is used to perform admission control. The idea is that by using very conservative admission control algorithms based on worst case analysis, together with peak rate policing at ingress nodes and static priority scheduling at core nodes, it is possible to ensure that all premium service packets incur very small queueing delay.

One important design question is: how conservative does the admission control algorithm need to be? In other words, what is the upper limit on the utilization of the network capacity that can be allocated to premium traffic if we want the premium service to achieve the same level of service assurance as the guaranteed service, such that the queueing delay of all premium service packets is bounded by a fixed number even in the worst case?

For the purpose of this discussion, we use *flow* to refer to a subset of packets that traverse the same path inside a Diffserv domain between two edge nodes. Thus, with the highest level of traffic aggregation, a flow consists of all packets between the same pair of ingress and egress nodes. Note that even in this case, the number of flows in a network can be quite large as it may increase quadratically with the number of edge nodes.

Let us consider a domain consisting of 4×4 routers with links of capacity C. Assume that the fraction of the link capacity allocated to the premium traffic is limited to γ. Assume also that all flows have equal packet sizes, and that each ingress node shapes not only each flow, but also the aggregate traffic at each of its outputs. Figure 14(a) shows the traffic pattern at the first core router along a path. Each input receives 12 identical flows, where each flow has a reservation of $\gamma C/12 = C/48$. Let τ be the transmission time of one packet, then as shown in the Figure, the inter-arrival time between two consecutive packets in the each flow is 48τ, and the inter-arrival time between two consecutive packets in the aggregate flow is 4τ.

Assume the first three flows at each input are forwarded to output 1. This will cause a burst of 12 packets to arrive output 1 in a 8τ long interval and the last packet of the burst to incur an *additional* delay of 3τ. Now assume that the next router receives at each input a traffic pattern similar to the one generated by output 1 of the first core router, as shown in Figure 14(b). In addition, assume that the *last* three flows from each input burst are forwarded to output 1. This will cause a burst of 12 packets to arrive output 1 in a 2τ long interval and the last packet in the

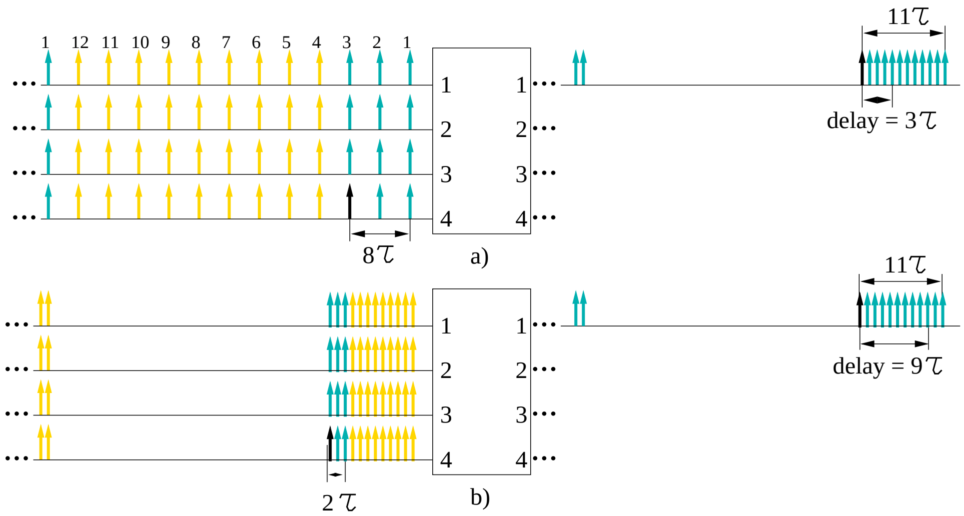

Figure 14: Per-hop worst-case delay experienced by premium traffic in a Diffserv domain. (a) and (b) shows the traffic pattern at the first and a subsequent node. The black and all dark grey packets go to the first output; the light grey packets go to the other outputs.

burst to incur an *additional* delay of 9τ. Thus, after two hops, a packet is delayed by as much as 12τ. This pattern can be repeated for all subsequent hops.

In general, consider a $k \times k$ router, and let n be the number of flows that traverse each link. For simplicity, assume that $\gamma \geq 1/k$. Then it can be shown that the worst case delay experienced by a packet after h hops is

$$D = \left(n - 1 - \left(\frac{n}{k} - 1\right)\frac{1}{\gamma}\right)\tau + (h-1)n\frac{k-1}{k}\tau + h\tau, \tag{23}$$

where the first term is the additional delay at the first hop, the second term is the additional delay at all subsequent hops, and the last term accounts for the packet transmission time at each hop. As a a numerical example, let $C = 1$ Gbps, a packet size of 1500 bytes, $k = 16$, $\gamma = 10\%$, $n = 1500$ and $h = 15$. From here we obtain $\tau = 12$ μsec, and a delay D of over 240 ms. Finally, if $\gamma < 1/k$, it can be shown that it will take only $\lceil \log_k(1/\gamma) \rceil$ hops to achieve a continuous burst. For example, for $\gamma = 1\%$ and $k = 16$, it takes only two hops to obtain a continuous burst.

The above example demonstrates that low network utilization and traffic shaping at ingress nodes alone are not enough to guarantee a "small" worst-case delay for *all* the premium traffic. This result is not surprising. Even using a per flow scheduler like Weighted Fair Queueing (WFQ), will not help to reduce the worst case end-to-end delay for *all* packets. In fact, if all flows in the above example are given the same weight, the worst case delay under WFQ is $hn\tau$, which is basically the same as the one given by Eq. (23). However, the major advantage of using WFQ is that it allows us to *differentiate* among flows, which is a critical property as long as we cannot guarantee a "small" delay to all flows. In addition, WFQ can achieve 100% utilization.

Appendix B: Proof of Theorem 1

In this appendix we show that a network of CJVC servers provides the same end-to-end delay guarantees as a network of Jitter-VC servers. In particular, in Theorem 1 we show that the deadline of a packet at the last hop in both systems is the same. This result is based on Lemmas 2 and 3 which give the expressions of the deadline of a packet at the last hop in a network of Jitter-VC, and a network of CJVC servers, respectively. First, we present a preliminary result used in proving Lemma 2.

Lemma 1 *Consider a network of Jitter-VC servers. Let π_j denote the propagation delay between hops j and $j+1$, and let τ_j be the maximum transmission time of a packet at node j. Then for any $j > 1$ and $i, k \geq 1$ we have*

$$d_{i,j+1}^k - d_{i,j}^k - \tau_j - \pi_j \geq d_{i,j}^k - d_{i,j-1}^k - \tau_{j-1} - \pi_{j-1}. \tag{24}$$

Proof. The proof is by induction on k. First, recall that by definition $g_{i,j}^k = d_{i,j}^k + \tau_j - s_{i,j}^k$ (see Table 1), and that for $j > 1$, $a_{i,j}^k = s_{i,j-1}^k + \pi_{j-1}$. From here and from Eqs. (1) and (2) we have then

$$d_{i,j}^k = \max(a_{i,j}^k + g_{i,j-1}^k, d_{i,j}^{k-1}) + \frac{l_i^k}{r_i} = \max(d_{i,j-1}^k + \tau_{j-1} + \pi_{j-1}, d_{i,j}^{k-1}) + \frac{l_i^k}{r_i}. \tag{25}$$

Basic Step. For $k = 1$ and any $j \geq 1$, from Eq. (25) we have trivially $d_{i,j}^1 = d_{i,j-1}^1 + \tau_{j-1} + \pi_{j-1} + l_i^1/r_i$, $\forall j > 1$, and therefore $d_{i,j}^1 - d_{i,j-1}^1 - \tau_{j-1} - \pi_{j-1} = l_i^1/r_i$, $\forall j > 1$.

Induction Step. Assume Ineq. (24) is true for k. Then we need to show that

$$\begin{aligned}
&d_{i,j+1}^{k+1} - d_{i,j}^{k+1} - \tau_j - \pi_j \geq d_{i,j}^{k+1} - d_{i,j-1}^{k+1} - \tau_{j-1} - \pi_{j-1} \Rightarrow \\
&\max(d_{i,j}^{k+1} + \tau_j + \pi_j, d_{i,j+1}^k) - \max(d_{i,j-1}^{k+1} + \tau_{j-1} + \pi_{j-1}, d_{i,j}^k) - \tau_j - \pi_j \geq \\
&\max(d_{i,j-1}^{k+1} + \tau_{j-1} + \pi_{j-1}, d_{i,j}^k) - \max(d_{i,j-2}^{k+1} + \tau_{j-2} + \pi_{j-2}, d_{i,j-1}^k) - \tau_{j-1} - \pi_{j-1},
\end{aligned} \tag{26}$$

where the second Inequality follows after using Eq. (25). Next consider two cases: whether $d_{i,j-1}^{k+1} + \tau_{j-1} + \pi_{j-1} \leq d_{i,j}^k$ or not. Assume $d_{i,j-1}^{k+1} + \tau_{j-1} + \pi_{j-1} \leq d_{i,j}^k$. From Ineq. (26) and from the induction hypothesis we obtain

$$\begin{aligned}
d_{i,j+1}^{k+1} - d_{i,j}^{k+1} - \tau_j - \pi_j \; &= \; \max(d_{i,j}^{k+1} + \tau_j + \pi_j, d_{i,j+1}^k) - \\
&\quad\; \max(d_{i,j-1}^{k+1} + \tau_{j-1} + \pi_{j-1}, d_{i,j}^k) - \tau_j - \pi_j
\end{aligned} \tag{27}$$

$$
\begin{aligned}
&= \max(d_{i,j}^{k+1} + \tau_j + \pi_j, d_{i,j+1}^{k}) - d_{i,j}^{k} - \tau_j - \pi_j \\
&\geq d_{i,j+1}^{k} - d_{i,j}^{k} - \tau_j - \pi_j \qquad \text{(induction hypothesis)} \\
&\geq d_{i,j}^{k} - d_{i,j-1}^{k} - \tau_{j-1} - \pi_{j-1} \\
&\geq d_{i,j}^{k} - \max(d_{i,j-2}^{k+1} + \tau_{j-1} + \pi_{j-1}, d_{i,j-1}^{k}) - \tau_{j-1} - \pi_{j-1} \\
&= \max(d_{i,j-1}^{k+1} + \tau_{j-1} + \pi_{j-1}, d_{i,j}^{k}) - \\
&\quad \max(d_{i,j-2}^{k+1} + \tau_{j-2} + \pi_{j-2}, d_{i,j-1}^{k}) - \tau_{j-1} - \pi_{j-1} \\
&= d_{i,j}^{k+1} - d_{i,j-1}^{k+1} - \tau_{j-1} - \pi_{j-1}.
\end{aligned}
$$

Next, assume that

$$
d_{i,j-1}^{k+1} + \tau_{j-1} + \pi_{j-1} > d_{i,j}^{k}. \tag{28}
$$

From here and by using Eq. (25) and Ineq. (26) we have

$$
\begin{aligned}
d_{i,j+1}^{k+1} - d_{i,j}^{k+1} - \tau_j - \pi_j &= \max(d_{i,j}^{k+1} + \tau_j + \pi_j, d_{i,j+1}^{k}) - \qquad (29) \\
&\quad \max(d_{i,j-1}^{k+1} + \tau_{j-1} + \pi_{j-1}, d_{i,j}^{k}) - \tau_j - \pi_j \\
&= \max(d_{i,j}^{k+1} + \tau_j + \pi_j, d_{i,j+1}^{k}) - d_{i,j-1}^{k+1} - \tau_{j-1} - \pi_{j-1} - \tau_j - \pi_j \\
&\geq d_{i,j}^{k+1} - d_{i,j-1}^{k+1} - \tau_{j-1} - \pi_{j-1} \\
&= d_{i,j}^{k+1} - \max(d_{i,j-1}^{k+1} + \tau_{j-1} + \pi_{j-1}, d_{i,j}^{k}) \qquad \text{(Eq. (25))} \\
&= \frac{l_i^{k+1}}{r_i} \qquad \text{(Eq. (25))} \\
&= d_{i,j-1}^{k+1} - \max(d_{i,j-2}^{k+1} + \tau_{j-2} + \pi_{j-2}, d_{i,j-1}^{k}) \\
&= \max(d_{i,j-1}^{k+1} + \tau_{j-1} + \pi_{j-1}, d_{i,j}^{k}) - \tau_{j-1} - \pi_{j-1} - \qquad \text{(Ineq. (28))} \\
&\quad \max(d_{i,j-2}^{k+1} + \tau_{j-2} + \pi_{j-2}, d_{i,j-1}^{k}) \\
&= d_{i,j}^{k+1} - d_{i,j-1}^{k+1} - \tau_{j-1} - \pi_{j-1}.
\end{aligned}
$$

This completes the proof. $\square$

Lemma 2 *The deadline of any packet p_i^k, $k > 1$, at the last hop h in a network of Jitter-VC servers is*

$$
d_{i,h}^{k} = \max\left(e_{i,1}^{k} + h\frac{l_i^k}{r_i} + \sum_{m=1}^{h-1}(\tau_m + \pi_m), d_{i,h}^{k-1} + \frac{l_i^k}{r_i}\right). \tag{30}
$$

Proof. Let $j^* > 1$ be the last hop for which $d_{i,j^*-1}^{k} + \tau_{j^*-1} + \pi_{j^*-1} < d_{i,j^*}^{k-1}$. We consider two cases whether j^* exists or not.

Case 1. (j^* does not exist) From Eq. (1) we have $e^k_{i,j} = d^k_{i,j-1} + \tau_{j-1} + \pi_{j-1}$, $\forall j > 1$. From here and by using Eq. (2) we obtain

$$d^k_{i,h} = e^k_{i,1} + h\frac{l^k_i}{r_i} + \sum_{m=1}^{h-1}(\tau_m + \pi_m). \tag{31}$$

Because we assume that j^* does not exist we also have $d^k_{i,h} = e^k_{i,h} + l^k_i/r_i \geq d^{k-1}_{i,h} + l^k_i/r_i$, which concludes the proof of this case.

Case 2. (j^* exists) In this case we show that $j^* = h$. Assume this is not true. Then we have $e^k_{i,j} = d^k_{i,j-1} + \tau_{j-1} + \pi_{j-1}$, $\forall j > j^*$. By using Eq. (2) we obtain

$$d^k_{i,h} = e^k_{i,j^*} + (h - j^* + 1)\frac{l^k_i}{r_i} + \sum_{m=j^*}^{h-1}(\tau_m + \pi_m). \tag{32}$$

On the other hand, by the definition of j^* and from Eqs. (1) and (2) we have

$$\begin{aligned} d^k_{i,j^*} &= \max(d^k_{i,j^*-1} + \tau_{j^*-1} + \pi_{j^*-1}, d^{k-1}_{i,j^*}) + \frac{l^k_i}{r_i} \\ &> d^k_{i,j^*-1} + \tau_{j^*-1} + \pi_{j^*-1} + \frac{l^k_i}{r_i}. \end{aligned} \tag{33}$$

As a result we obtain $d^k_{i,j^*} - d^k_{i,j^*-1} - \tau_{j^*-1} - \pi_{j^*-1} > l_i/r_i$. By iteratively applying Lemma 1 we have

$$d^k_{i,m+1} - d^k_{i,m} - \tau_m - \pi_m \geq d^k_{i,j^*} - d^k_{i,j^*-1} - \tau_{j^*-1} - \pi_{j^*-1} > \frac{l^k_i}{r_i}, \quad \forall m \geq j^* \tag{34}$$

From Ineq. (34) we obtain

$$\sum_{m=j^*}^{h-1}(d^k_{i,m+1} - d^k_{i,m} - \tau_m - \pi_m) \geq (h - j^*)(d^k_{i,j^*} - d^k_{i,j^*-1} - \tau_{j^*-1} - \pi_{j^*-1}) > (h - j^*)\frac{l^k_i}{r_i}, \tag{35}$$

where the right-hand side term can be expressed as

$$\sum_{m=j^*}^{h-1}(d^k_{i,m+1} - d^k_{i,m} - \tau_m - \pi_m) = d^k_{i,h} - d^k_{i,j^*} - \sum_{m=j^*}^{h-1}(\tau_m + \pi_m). \tag{36}$$

By combining Ineq. (35) and Eq. (36) we get

$$\begin{aligned} d^k_{i,h} &> d^k_{i,j^*} + (h - j^*)\frac{l^k_i}{r_i} + \sum_{m=j^*}^{h-1}(\tau_m + \pi_m) \\ &= e^k_{i,j^*} + (h - j^* + 1)\frac{l^k_i}{r_i} + \sum_{m=j^*}^{h-1}(\tau_m + \pi_m). \end{aligned} \tag{37}$$

But this inequality contradicts Eq. (32) and therefore proves our statement, i.e., $j^* = h$. Thus, $e_{i,h}^k = d_{i,h}^{k-1}$. From here and from Eqs. (1) and (2) we get

$$d_{i,h}^k = e_{i,h}^k + \frac{l_i}{r_i} = d_{i,h}^{k-1} + \frac{l_i}{r_i}. \tag{38}$$

Now, from Eq. (1) it follows trivially that

$$e_{i,j}^k = \max(d_{i,j-1}^k + \tau_{j-1} + \pi_{j-1}, d_{i,j-1}^k) \geq d_{i,j-1}^k + \tau_{j-1} + \pi_{j-1}, \quad j \geq 1. \tag{39}$$

By iterating over the above equation and then using Eq. (2) we get

$$d_{i,h}^k \geq e_{i,1}^k + h\frac{l_i^k}{r_i} + \sum_{m=1}^{h-1}(\tau_m + \pi_m), \tag{40}$$

which together with Eq. (38) lead us to Eq. (30).

This completes the proof of the lemma. □

Lemma 3 *The deadline of any packet p_i^k, $k > 1$, at the last hop h in a network of CJVC servers is*

$$d_{i,h}^k = \max\left(e_{i,1}^k + h\frac{l_i^k}{r_i} + \sum_{m=1}^{h-1}(\tau_m + \pi_m), d_{i,h}^{k-1} + \frac{l_i^k}{r_i}\right). \tag{41}$$

Proof. We consider two cases whether $\delta_i^k = 0$ or not.

Case 1. ($\delta_i^k = 0$) From Eqs. (2) and (6) it follows that

$$d_{i,h}^k = e_{i,1}^k + h\frac{l_i^k}{r_i} + \sum_{m=1}^{h-1}(\tau_m + \pi_m). \tag{42}$$

On the other hand, by the definition of δ_i^k (see Ineq. (3) and Eq. (4)) we have $e_{i,j}^k = d_{i,j-1}^k + \tau_{j-1} + \pi_{j-1} + \delta_i^k \geq d_{i,j}^{k-1}$, $\forall j > 1$. From here and from Eq. (2) we obtain

$$d_{i,h}^k \geq d_{i,h}^{k-1} + \frac{l_i^k}{r_i}. \tag{43}$$

From this inequality and Eq. (42), Eq. (41) follows.

Case 2. ($\delta_i^k > 0$) By using Eqs. (2) and (10) we obtain

$$
\begin{aligned}
d_{i,h}^k &= e_{i,1}^k + h\frac{l_i^k}{r_i} + (h-1)\delta_i^k + \sum_{m=1}^{h-1}(\tau_m + \pi_m) \qquad (44)\\
&= e_{i,1}^k + h\frac{l_i^k}{r_i} + \left((h-1)\delta_i^{k-1} + (h-1)\frac{l_i^{k-1} - l_i^k}{r_i} - e_{i,1}^k + e_{i,1}^{k-1} + \frac{l_i^{k-1}}{r_i}\right) \\
&\quad \sum_{m=1}^{h-1}(\tau_m + \pi_m) \\
&= e_{i,1}^{k-1} + h\frac{l_i^{k-1}}{r_i} + (h-1)\delta_i^{k-1} + \sum_{m=1}^{h-1}(\tau_m + \pi_m) + \frac{l_i^k}{r_i} \\
&= d_{i,h}^{k-1} + \frac{l_i^k}{r_i}.
\end{aligned}
$$

Since $\delta_i^k > 0$, by using again Eq. (2) and (7) we get

$$
\begin{aligned}
d_{i,h}^k &= e_{i,1}^k + h\frac{l_i^k}{r_i} + (h-1)\delta_i^k + \sum_{m=1}^{h-1}(\tau_m + \pi_m) \qquad (45)\\
&> e_{i,1}^k + h\frac{l_i^k}{r_i} + \sum_{m=1}^{h-1}(\tau_m + \pi_m).
\end{aligned}
$$

which together with Eq. (44) lead to Eq. (41). □

Theorem 1 *The deadlines of a packet at the last hop in a network of CJVC servers is equal to the deadline of the same packet in a corresponding network of Jitter-VC servers.*

Proof. From Eqs (1) and (2) it is easy to see that in a network of Jitter-VC servers we have

$$
d_{i,h}^1 = e_{i,1}^1 + h\frac{l_i^k}{r_i} + \sum_{m=1}^{h-1}(\tau_m + \pi_m). \qquad (46)
$$

Similarly, in a network of CJVC servers, from Eqs. (1) and (7), and by using the fact that $\delta_i^1 = 0$ (see Eq. 8), we obtain an identical expression for $d_{i,h}^1$ (i.e., Eq. (46)).

Finally, since (a) the eligible times of all packets p_i^k at the first hop, i.e., $e_{i,1}^k$ ($\forall k \geq 1$), are identical for both Jitter-VC and CJVC servers, and since (b) the deadlines of the packets at the last hop, i.e., $d_{i,h}^k$ ($\forall k \geq 1$), are computed based on the same formulae (see Eqs. (30), (41) and 46), it follows that $d_{i,h}^k$, ($\forall k \geq 1$) are identical in both a network of Jitter-VC, and a network of CJVC servers. □

Appendix C: Proof of Theorem 2

To prove Theorem 2 (see Section 3.3) we prove two intermediate results: Lemma 7 which gives the buffer occupancy for the case when all flows have identical rates, and Lemma 10 which gives the buffer occupancy for arbitrary flow rates.

Identical Flow Rates

Consider a work-conserving server with the output rate one, which is traversed by n flows with identical reservations of $1/n$. Assume that the time axis is divided in *unit* sized slots, where slot t corresponds to the time interval $[t, t+1)$. Assume that at most one packet can be sent during each slot, i.e., the packet transmission time is one time unit. Finally, assume that the starting times of the backlogged periods of any two flows are uncorrelated. In practice, we enforce this by delaying the first packet of a backlogged period by an amount drawn from an uniform distribution in the range $[t_{arrival}, t_{arrival} + n)$, where $t_{arrival}$ is the arrival time of the first packet in the backlogged period. Note that according to Eq. (1), packets' eligible times during a flow backlogged interval are *periodic* with period n. Thus, without loss of generality, we assume that the arrival process of any flow during a backlogged interval is periodic.

Let $r(t', t'')$ denote the number of packets received (i.e., became eligible) during the interval $[t', t'')$, and let $s(t', t'')$ denote the number of packets sent during the same interval. Note that $r(t', t'')$ and $s(t', t'')$ do *not* include packets received/transmitted during slot t''. Let $q(t)$ denote the size of the queue at the *beginning* of slot t. Then, if no packets are dropped, we have

$$q(t'') = q(t') + r(t', t'') - s(t', t''). \tag{47}$$

Since at most one packet is sent during each time slot, we have $s(t', t'') \leq t'' - t'$. The inequality holds when $[t', t'')$ belongs to a server busy period. A busy period is defined as an interval during which server's queue is never empty. Also, note that if t' is the starting time of a busy period $q(t') = 0$.

The next result shows that to compute an upper bound for $q(t)$ it is enough to consider only the scenarios in which all flows are continuously backlogged.

Lemma 4 *Let t_1 be an arbitrary time slot during a server busy period that starts at time t_0. Assume flow i is not continuously backlogged during the interval $[t_0, t_1)$. Then $q(t_1)$ can only increase if flow i becomes continuously backlogged during $[t_0, t_1)$.*

Proof. Consider two cases whether flow i is idle during the entire interval $[t_0, t_1)$, or not.

If flow i is idle during $[t_0, t_1)$, consider the modified scenario in which flow i becomes backlogged at an arbitrary time $t < t_0$, and remains continuously backlogged during $[t_0, t_1)$. In addition, assume that the arrival patterns of all the other flows remain unchanged. As a result, it is easy to see that in the modified scenario the total number of packets received during $[t_0, t_1)$ can only increase, while the starting time of the busy interval can only decrease. Let r', s', and q' denote the corresponding values in the modified scenario. Then $q'(t_0) \geq q(t_0) = 0$, $r'(t_0, t_1) \geq r(t_0, t_1)$, and $s'(t_0, t_1) = s(t_0, t_1) = t_1 - t_0$. From Eq. (47) it follows then that $q'(t_1) \geq q(t_1)$.

In the second case, when flow i is neither idle nor continuously backlogged during the interval $[t_0, t_1)$, let t' denote the time when the last packet of flow i arrives during $[t_0, t_1)$. Next consider the modified scenario in which flow i's packets arrive at times: $t' - na, \ldots, t' - n, t', t' + n, \ldots, t' + nb$, such that $t' - na \leq t_0$, and $t_1 \leq t' + nb$. It is easy to see then that the number of packets of flow i that arrive during $[t_0, t_1)$ is no smaller than the number of packets of flow i that arrive during the same interval in the original scenario. By assuming that the arrival patterns of all the other flows do not change, it follows that $r'(t_0, t_1) \geq r(t_0, t_1)$. In addition, since at most $t_1 - t_0$ packets are transmitted during $[t_0, t_1)$ we have $s'(t_0, t_1) \leq t_1 - t_0$. The inequality holds if, after changing the arrival pattern of flow i, the server is no longer busy during the entire interval $[t_0, t_1)$. In addition, we have $q'(t_0) \geq 0$, and from the hypothesis $q(t_0) = 0$. Finally, from Eq. (47) we obtain $q'(t_1) \geq q(t_1)$, which concludes the proof of the lemma. □

In consequence in the remaining of this section we limit our study to a busy period in which all flows are continuously backlogged.

Let t_1 be the time when the last flow becomes backlogged. Let t_0 be the latest time no larger than t_1 when the server become busy, i.e., it has no packet to send during $[t_0 - 1, t_0)$ and is continuously busy during the interval $[t_0, t_1 + 1)$. Then we have the following result.

Lemma 5 *If all flows remain continuously backlogged after time t_1, the server is busy for any time $t \geq t_0$.*

Proof. By the definition of t_0, the server is busy during $[t_0, t_1)$. Next we show that the server is also busy for any $t_1 \geq 0$.

Consider a flow that becomes backlogged at time t', Since its arrival process is periodic it follows that during any interval $[t' - n + i, t' + i)$, $\forall i > 0$, exactly one packet of this flow arrives. Since after time t_1 all n flows are backlogged, exactly n packets are received during $[t_1 - n + i, t_1 + i)$, $\forall i > 0$. Since at most n packets are sent during each of these intervals it follows that the server cannot be idle during any slot i. □

Consider a buffer of size s. Our goal is to compute the probability with which the buffer overflows during an arbitrary interval $[t_0, t_0 + d)$. From Lemma 5 it follows that since the server is busy during $[t_0, t_0 + d)$, exactly d packets are transmitted during this interval. In addition, since the starting times of flows' backlogged periods are not correlated, in the followings we also assume that the starting times of a flow's backlogged period is *not* correlated with the starting time, t_0, of a busy period. Thus, during the interval $[t_0, t_0+d)$, a flow receives $\lceil d/n \rceil$ packets with probability $p(d) = d/n - \lfloor d/n \rfloor$, and $\lfloor d/n \rfloor$ with probability $1-p$. Since this probability is periodic with period n it will suffice to consider only intervals of size at most n. Consequently, in the followings we assume $d \leq n$. The probability to receive one packet during $[t_0, t_0+d)$ is then

$$p(d) = \frac{d}{n}. \tag{48}$$

Let $p(m; d)$ denote the probability with which exactly m packets are received during the time interval $[t_0, t_0+d)$, where

$$p(m; d) = \binom{n}{m} p(d)^m (1-p(d))^{n-m}. \tag{49}$$

Now, let $P(x > s, u)$ denote the probability with which the queue size exceeds s *at* time t_0+u. Since the server is idle at t_0 and busy during $[t_0, t_0+u)$, from Eq. (47) follows that the server's queue overflows when more than $u+s$ packets are received during $[t_0, t_0+u)$. Thus, we have

$$P(x > s, u) = \sum_{i=u+s+1}^{n} p(i; u) = \sum_{i=u+s+1}^{n} \binom{n}{i} p(u)^i (1-p(u))^{n-i}. \tag{50}$$

The next result computes $P(x > s, u)$.

Lemma 6 *The probability that a queue of size s overflows at time $t_0 + u$ is bounded by*

$$P(x > s, u) < \beta(n)\sqrt{\frac{1}{2\pi}} \left(\frac{1-(s-1)/2n}{1+(s-1)/2n}\right)^{2s} \frac{(n+s)^2}{4sn}. \tag{51}$$

where $\beta(n) = (n/e)^{1+(1/12n)}$.

Proof. From Eq. (49) we obtain

$$p(m+1; u) = \frac{p(u)}{1-p(u)} \cdot \frac{n-m}{m+1} \cdot p(m; u), \tag{52}$$

By plugging the above equation and Eq. (48) into Eq. (50) we obtain

$$\begin{aligned} P(x > s, u) &= p(u+s; u) \sum_{i=u+s+1}^{n} \left(\prod_{k=u+s}^{i-1} \frac{n-k}{k+1}\right) \left(\frac{u}{n-u}\right)^{i-u-s} \\ &< p(u+s; u) \sum_{i=u+s+1}^{n} \left(\prod_{k=u+s}^{i-1} \frac{n-u-s}{u+s}\right) \left(\frac{u}{n-u}\right)^{i-u-s} \\ &= p(u+s; u) \sum_{i=u+s+1}^{n} \left(\frac{n-u-s}{u+s} \cdot \frac{u}{n-u}\right)^{i-u-s} \end{aligned} \tag{53}$$

Next, it can be easily verified that for any positive reals a, b, and x, such that $b - x \geq 0$, we have

$$\frac{a}{a+x} \cdot \frac{b-x}{b} \leq \left(\frac{a+b-x}{a+b+x}\right)^2. \tag{54}$$

By taking $a = u$, $b = n - u$, $x = s$, Eq. (53) becomes

$$\begin{aligned} P(x > s, u) &< p(u+s; u) \sum_{i=u+s+1}^{n} \left(\frac{n-s}{n+s}\right)^{2(i-u-s)} < p(u+s;u) \sum_{i=0}^{\infty} \left(\frac{n-s}{n+s}\right)^{2i} \\ &< p(u+s;u) \frac{(n+s)^2}{4sn}. \end{aligned} \tag{55}$$

Next, it remains to bound $p(u+s;u)$. From Eqs. (48) and (52) we have

$$p(u+s;u) = p(u;u) \prod_{i=0}^{s-1} \left(\frac{u}{n-u} \cdot \frac{n-u-i}{u+i+1}\right) < \prod_{i=0}^{s-1} \left(\frac{u}{u+i} \cdot \frac{n-u-i}{n-u}\right). \tag{56}$$

By using Ineq. (54) with $a = u$, $b = n - u$, and $x = i$, we obtain

$$p(u+s;u) = p(u;u) \prod_{i=0}^{s-1} \left(\frac{n-i}{n+i}\right)^2 = p(u;u) \prod_{i=0}^{s-1} \left(\frac{n-i}{n+i} \cdot \frac{n-s+1+i}{n+s-1-i}\right). \tag{57}$$

Again, by applying Ineq. (54) to the pairs $(n-i)/(n+i)$ and $(n-s+1+i)/(n+s-1-i)$, $\forall i < s/2$, we have

$$p(u+s;u) < p(u;u) \left(\frac{2n-(s-1)}{2n+(s-1)}\right)^{2s} = p(u;u) \left(\frac{1-(s-1)/2n}{1+(s-1)/2n}\right)^{2s}. \tag{58}$$

To bound $p(u;u)$ we use Stirling inequalities [8], i.e., $\sqrt{2\pi n}(n/e)^n < n! < \sqrt{2\pi n}(n/e)^{n+(1/12n)}$, $\forall n \geq 1$. From here we have

$$\begin{aligned} \binom{n}{n-u} &< \frac{\sqrt{2\pi n}(n/e)^{n+(1/12n)}}{\sqrt{2\pi u}(u/e)^u \sqrt{2\pi(n-u)}((n-u)/e)^{n-u}} \\ &= \sqrt{\frac{n}{2\pi(n-u)u}} \cdot \frac{n^n (n/e)^{1/12n}}{u^u (n-u)^{n-u}}. \end{aligned} \tag{59}$$

By combining Eqs. (48), (49) and (59), we obtain

$$p(u;u) < \beta(n) \sqrt{\frac{n}{2\pi u(n-u)}} \leq \beta(n) \sqrt{\frac{1}{2\pi}}. \tag{60}$$

where $\beta(n) = (n/e)^{1+(1/12n)}$ and the last inequality follows from the fact that $n/((n-u)u) < 1$, for any $u \geq 1$, $n \geq 2$. By plugging the above result in Eq. (55) we obtain

$$P(x > s, u) < \beta(n)\sqrt{\frac{1}{2\pi}}\left(\frac{1-(s-1)/2n}{1+(s-1)/2n}\right)^{2s}\frac{(n+s)^2}{4sn}. \tag{61}$$

□

Lemma 7 *Consider n flows with identical rates and unit packet sizes. Then given a buffer of size s, were*

$$s \geq \sqrt{n\left(\frac{\ln n}{2} - \frac{\ln \varepsilon}{2} - 1\right)}, \tag{62}$$

the probability that the buffer overflows during an arbitrary time slot when the server is busy is asymptotically $< \varepsilon$.

Proof. To compute the asymptotic bound for $P(x > s, u)$ assume that $s \ll n$. Since $(1-x)/(1+x) \simeq 1-2x$ and $\ln(1-x) \simeq x$, for $x \to 0$, and since $(n+s)^2/sn < n$ for $n > s \geq 4$, and $\beta(n) < 1.102$ for any $n \geq 1$, by using Eq. (51) we obtain[7]

$$\begin{aligned} \ln P(x > s, u) &\simeq \ln\left(\beta(n)\sqrt{\frac{1}{2\pi}}\right) + 2s \cdot \ln\left(\frac{1-(s-1)/2n}{1+(s-1)/2n}\right) + \ln n - \ln 4 \\ &\simeq \ln\left(\beta(n)\sqrt{\frac{1}{2\pi}}\right) + 2s \cdot \ln\left(1 - \frac{s-1}{n}\right) + \ln n - \ln 4 \\ &\simeq \ln\left(\beta(n)\sqrt{\frac{1}{2\pi}}\right) - 2s\frac{s-1}{n} + \ln n - \ln 4 \\ &< -2 - 2\frac{s(s-1)}{n} + \ln n \simeq 2(-1 - \frac{s^2}{n}) + \ln n. \end{aligned} \tag{63}$$

Using ε to bound $P(x > s, u)$ leads us to

$$\begin{aligned} &P(x > s; u) \leq \varepsilon \Rightarrow \\ &2\left(-1 - \frac{s^2}{n}\right) + \ln n \leq \ln \varepsilon \Rightarrow \\ &s \geq \sqrt{n\left(\frac{\ln n}{2} - \frac{\ln \varepsilon}{2} - 1\right)}. \end{aligned} \tag{64}$$

□

[7] More precisely $\ln \beta(n)\sqrt{1/(2\pi)} - \ln 4 \leq -2.2081062\ldots$.

Next we prove a stronger result by computing an asymptotic upper bound for the probability with which a queue of size s overflows during an *arbitrary* busy interval. Let $Q(x > s)$ denote this probability. The key observation is that since all flows have period n, the aggregate arrival traffic will have the same period n. In addition, since during each of these periods exactly n packets are received/transmitted it follows that the queue size at any time $t_0 + i \cdot n + j$ is the same, $\forall i, j \geq 0$. Consequently, if the queue does not overflow during $[t_0, t_0 + n)$, the queue will not overflow at any other time $t \geq t_1$ during the same busy period. Thus, the problem reduces to compute the probability of queue overflowing during the interval $[t_0, t_0 + n)$. Then we have the following result.

Corollary 1 *Consider n flows with identical rates and unit packet sizes. Then given a buffer of size s, were*

$$s \geq \sqrt{n(\ln n - (\ln \varepsilon)/2 - 1)}, \tag{65}$$

the probability that the buffer overflows during an arbitrary busy interval is asymptotically $< \varepsilon$.

Proof. Let ε' be the probability that a buffer of size s overflows at an instant t during the busy interval $[t_0, t_0 + u)$. Then the probability that the buffer overflows during this interval is smaller than $1 - (1 - \varepsilon')^u < u \cdot \varepsilon'$. Now, recall that if the buffer does not overflow during $[t_0, t_0 + n)$, the buffer will not overflow after time $t_0 + n$. Thus the probability that the buffer will not overflow during an arbitrary busy period is less than $n\varepsilon'$. Finally, let $\varepsilon = n \cdot \varepsilon'$, and apply the result of Lemma 7 for ε', i.e.,

$$s \;\; \geq \;\; \sqrt{n\left(\frac{\ln n}{2} - \frac{\ln(\varepsilon/n)}{2} - 1\right)} = \sqrt{n\left(\ln n - \frac{\ln \varepsilon}{2} - 1\right)}. \tag{66}$$

□

Arbitrary Flow Rates

In this section we determine the buffer bound for a system in which packets are of unit size, but the reservations can be arbitrary. The basic idea is to use a succession of transformations to reduce the problem to the case in which the probabilities associated to the flows can take at most three distinct values, and then to apply the results from the previous case when all reservations are assumed to be identical.

Consider n flows, and let r_k denote the rate reserved by flow k, where

$$\sum_{k=1}^{n} r_k = 1. \tag{67}$$

Consider again the case when all flows are continuously backlogged. Let t_0 denote the starting time of a busy period. Since the time when flow k becomes backlogged is assumed to be independent of t_0, it follows that during the interval $[t_0, t_0 + d)$ flow k receives exactly $\lfloor d \cdot r_k \rfloor + 1$ packets with probability

$$p_k(d) = d \cdot r_k - \lfloor d \cdot r_k \rfloor, \tag{68}$$

and $\lfloor d \cdot r_k \rfloor$ packets with probability $1 - p_k(d)$.

Let $p(m; d)$ denote the probability with which the server receives exactly $\sum_{k=1}^{n} \lfloor d \cdot r_k \rfloor + m$ packets during the interval $[t_0, t_0 + d)$. Then

$$p(m; d) = T_n^m(p_1(d), p_2(d), \ldots, p_n(d)), \tag{69}$$

where $T_n^m(p_1(d), p_2(d), \ldots, p_n(d))$ is the coefficient of x^m in the expansion of

$$\prod_{i=1}^{n} (x p_i(d) + (1 - p_i(d))). \tag{70}$$

Note that when all flows have equal reservations, i.e., $r_k = 1/n, 1 \le k \le n$, Eq. (69) reduces to Eq. (49).

By using Eq. (68) the number of packets received during $[t_0, t_0 + d)$ can be written as

$$\sum_{k=1}^{n} \lfloor d \cdot r_k \rfloor + m = \sum_{k=1}^{n} (d \cdot r_k - p_k(d)) + m = d - \sum_{k=1}^{n} p_k(d) + m. \tag{71}$$

Since t_0 is the starting time of the busy period and since the server remains busy during $[t_0, t_0 + d)$, from Eq. (47) it follows that $q(t_0 + d) = m - \sum_{k=1}^{n} p_k(d)$.

Similarly, the probability $P(x > s, u)$ to overflow a queue of size s at time $t_0 + u$ is

$$P(x > s, u) = \sum_{i=v+1}^{n} p(i; u), \tag{72}$$

where $v = \sum_{k=1}^{n} p_k(u) + s$.

Since in the followings $p_k(u)$ is always defined over $[t_0, t_0+u)$ we will drop the argument from the $p_k(u)$'s notation. Next, note that for any two flows k and l, $p(m;u)$ can be rewritten as

$$p(m;u) = p_k p_l A_{k,l}(m) + (p_k(1-p_l) + (1-p_k)p_l) B_{k,l}(m) + (1-p_k)(1-p_l) C_{k,l}(m), \tag{73}$$

where $p_k p_l A_{k,l}$, represents all terms in $T_n^m(p_1, p_2, \ldots, p_n)$ that contain $p_k p_l$, $(p_k(1-p_l) + (1-p_k)p_l)B_{k,l}$ represents all terms that contain either $p_k(1-p_l)$ or $(1-p_k)p_l$, and $(1-p_k)(1-p_l)C_{k,l}$ represents all terms that contain $(1-p_k)(1-p_l)$.

From Eqs. (72) and (73), the probability to overflow a queue of size s at time $t_0 + u$ is then

$$\begin{aligned} P(x > s, u) &= \sum_{i=v+1}^{n} p(i;u) \qquad (74) \\ &= p_k p_l \cdot \mathcal{A}_{k,l}(v,n) + (p_k(1-p_l) + (1-p_k)p_l) \cdot \mathcal{B}_{k,l}(v,n) + \\ &\quad (1-p_k)(1-p_l) \cdot \mathcal{C}_{k,l}(v,n), \end{aligned}$$

where $\mathcal{A}_{k,l}(v,n) = \sum_{i=v+1}^{n} A_{k,l}(i)$, $\mathcal{B}_{k,l}(v,n) = \sum_{i=v+1}^{n} B_{k,l}(i)$, and $\mathcal{C}_{k,l}(v,n) = \sum_{i=v+1}^{n} C_{k,l}(i)$, respectively.

Our next goal is to reduce the problem of bounding $P(x > s, u)$ to the case in which the flows' probabilities take a limited number of values. This makes possible to use the results from the homogeneous reservations case without compromising too much the bound quality. The idea is to iteratively modify the values of the flows' probabilities, without decreasing $P(x > s, u)$. In particular, we consider the following simple transformation: select two probabilities p_k and p_l and update them as follows:

$$\begin{aligned} p'_k &= p_k - \delta, \qquad (75) \\ p'_l &= p_l + \delta, \end{aligned}$$

where δ is a real value such that $0 \le p'_l, p'_k \le 1$, and the new computed probability

$$\begin{aligned} P'(x > s, u) &= p'_k p'_l \cdot \mathcal{A}_{k,l}(v,n) + (p'_k(1-p'_l) + (1-p'_k)p'_l) \cdot \mathcal{B}_{k,l}(v,n) + \qquad (76) \\ &\quad (1-p'_k)(1-p'_l) \cdot \mathcal{C}_{k,l}(v,n). \end{aligned}$$

is greater or equal to $P(x > s, u)$.

It is interesting note that performing transformation (75) is equivalent to defining a new system in which the reservations of flows k and l are changed to r'_k and r'_l, respectively, such that $p'_k = d \cdot r'_k - \lfloor d \cdot r'_k \rfloor$, and $p'_l = d \cdot r'_l - \lfloor d \cdot r'_l \rfloor$. There are two observations worth noting about this system. First, by choosing $r'_k = r_k - \delta/d$ and $r'_l = r_l + \delta/d$ we maintain the invariant $\sum_{i=1}^{n} r_i = 1$.

Second, while in the new system the start time t_0 of the busy period may change, this will not influence $P'(x > s, u)$ as this depends only on the length of the interval $[t_0, t_0 + u)$.

Next, we give the details of our transformation. From Eqs. (74), (75) and (76), after some simple algebra, we obtain

$$P'(x > s, u) - P(x > s, u) = \delta(p_k - p_l - \delta)\mathcal{D}_{k,l}(v, n), \tag{77}$$

where

$$\mathcal{D}(i, j) = \mathcal{A}_{k,l}(i, j) - 2\mathcal{B}_{k,l}(i, j) + \mathcal{C}_{k,l}(i, j)). \tag{78}$$

Recall that our goal is to chose δ such that $P'(x > s, u) \geq P(x > s)$. Without loss of generality assume that $p_k > p_l$. We consider two cases: (1) if $\mathcal{D}_{k,l}(v, n) > 0$, then $\delta \geq 0$ and $p_k \geq p_l + \delta$ ($\delta < 0$ and $p_k < p_l + \delta$ cannot be simultaneously true); (2) if $\mathcal{D}_{k,l}(v, n) \leq 0$, then either $\delta \geq 0$ and $p_k \leq p_l + \delta$, or $\delta < 0$ and $p_k > p_l + \delta$.

Let $p_{min} = \min_{1 \leq i \leq n} p_i$, and $p_{max} = \max_{1 \leq i \leq n} p_i$, respectively. Consider the following three subsets, denoted U, V, and M, where U contains all flows k such that $p_k = p_{min}$, V contains all flows k such that $p_k = p_{max}$, and M contains all the other flows. The idea is then to successively apply the transformation (75) on $p_1, p_2, \ldots, p_n$, until the probabilities of all flows in M become equal. In this way we reduce the problem to the case in which the probabilities p_k can take at most three distinct values: p_{min}, p_{max}, and p_M, where $p_k = p_M$, $\forall k \in M$. Figure 15 shows the iterative algorithm to achieve this. Lemmas 8 and 9 prove that by using the algorithm in Figure 15, $p_1, p_2, \ldots, p_n$ converge asymptotically to the three values.

while ($|M| > 1$) **do** /* *while size of M is greater than one* */
  $p_l = \min_{i \in M}(p_i)$;
  $p_k = \max_{i \in M}(p_i)$;
  if ($D_{k,l}(v, n) > 0$)
    $p_k = p_l = (p_k + p_l)/2$;
  else
    $\delta = \max(p_k - p_{max}, p_{min} - p_l)$;
    $p_k = p_k - \delta$; $p_l = p_l + \delta$;
    if ($p_l = p_{min}$)
      $M = M \setminus \{l\}$; $U = U \cup \{l\}$;
    if ($p_k = p_{max}$)
      $M = M \setminus \{k\}$; $V = V \cup \{k\}$;

Figure 15: *Reducing $p_1, p_2, \ldots p_n$ to three distinct values.*

Lemma 8 *After an iteration of the algorithm in Figure 15 either the size of M decreases by one, or the standard deviation of the probabilities in M decreases by a factor of at least $(1 - \frac{1}{2|M|})$.*

Proof. The first part is trivial; if $D_{k,l}(v,n) \leq 0$ the size of M decreases by one. For the second part, let $\overline{p}$ denote the average values of probabilities associated to the flows in M, i.e.,

$$\overline{p} = \frac{\sum_{i \in M} p_i}{|M|}. \tag{79}$$

The standard deviation associated to the probabilities in M is

$$dev = \sum_{i \in M} (p_i - \overline{p})^2. \tag{80}$$

After averaging probabilities p_k and p_l, standard deviation v changes to

$$dev' = dev + 2\left(\frac{p_k + p_l}{2} - \overline{p}\right)^2 - (p_k - \overline{p})^2 - (p_l - \overline{p})^2 = dev - \frac{(p_k - p_l)^2}{2}. \tag{81}$$

Since p_k and p_l are the lowest, and respectively, the highest probabilities in M we have $(p_i - \overline{p})^2 \leq (p_l - p_k)^2$, $\forall i \in M$. From here and from Eqs. (80) and (81) we have

$$dev = \sum_{i \in M} (p_i - \overline{p})^2 \leq |M|(p_l - p_k)^2 = 2|M|(dev - dev') \Rightarrow dev' \leq dev \cdot \left(1 - \frac{1}{2|M|}\right). \tag{82}$$

□

Lemma 9 *Consider n flows, and let p_i denote the probability associated to flow i. Then, by using the algorithm in Figure 15, the probabilities p_i $(1 \leq i \leq n)$ converge to at most three values.*

Proof. Let ε be an arbitrary small real. The idea is then to show that after a finite number of iterations of the algorithm in Figure 15, the standard deviation of p_i's $(i \in M)$ becomes smaller than ε.

The standard deviation for the probabilities of flows in M is trivially bounded as follows

$$dev = \sum_{i \in M} (p_i - \overline{p})^2 \leq \sum_{i \in M} (p_{max} - p_{min})^2 = |M|(p_{max} - p_{min})^2 < n(p_{max} - p_{min})^2. \tag{83}$$

Assume $D_{k,l}(v,n) > 0$ (i.e., M does not change) for n_1 consecutive iterations. Then, by using Lemma 8, it is easy to see that n_1 is bounded above by N, where

$$dev \cdot \left(1 - \frac{1}{2|M|}\right)^N < dev \cdot \left(1 - \frac{1}{2n}\right)^N = \varepsilon \Rightarrow N = \frac{\ln(\varepsilon/dev)}{\ln(1 - 1/(2n))}. \tag{84}$$

Since the above bound, N, holds for any set M, it follows that after nN iterations, we are guaranteed that either set M becomes empty, case in which lemma is trivially true, or $dev < \varepsilon$. $\Box$

Thus, we have reduced the problem to compute an upper bound for probability $P(x > s, u)$ in a system in which probabilities take only three values at time u: p_{min}, p_{max}, and p_M.

Next we give the main result of this section

Lemma 10 *Consider n flows with unit packet sizes and arbitrary flow reservations. Then given a buffer of size s, were*

$$s \geq \sqrt{3n\left(\frac{\ln n}{2} - \frac{\ln \varepsilon}{2} - 1\right)}, \tag{85}$$

the probability that the buffer overflows in an arbitrary time slot during a server busy period is asymptotically $< \varepsilon$.

Proof. Consider the probability, $P(x > s, u)$, with which the queue overflows at time $t_0 + u$ (see Eq. (72)). Next, by using the algorithm in Figure 15, we reduce probabilities p_i's ($1 \leq i \leq n$) to three values: p_{min}, p_{max}, and p_M, respectively. Let p_i^f denote the final probability of flow i, and let $P^f(x > s, u)$ denote the final probability of the queue overflowing at time $t_0 + u$. More precisely, from Eqs. (72) and (69) we have

$$P^f(x > s, u) = \sum_{i=v+1}^{n} p^f(i; u) = \sum_{i=v+1}^{n} T_n^i(p_1, p_2, \ldots, p_n), \tag{86}$$

where $v = \sum_{k=1}^{n} p_k(u) + s$, and $p_i = p_{min}$, $\forall i \in U$, $p_i = p_{max}$, $\forall i \in V$, and $p_i = p_M$, $\forall i \in V$. Since after each transformation $P(x > s, u)$ can only increase, we have $P^f(x > s, u) \geq P(x > s, u)$.

Let n_U, n_V, and n_M be the number of flows in sets U, V, and M, respectively. Define integers v_u, v_V, and v_M, such that $v = v_u + v_V + v_M$, and $v_U < n_U, v_V < n_V$, and $v_M < n_M$, respectively. Then, it can be shown that

$$P^f(x > s, u) \;\; \leq \;\; P_U + P_V + P_M, \tag{87}$$

where

$$\begin{aligned} P_U &= \sum_{i=v_U+1}^{n_U} \binom{n_U}{i} p_{min}^i (1 - p_{min})^{n_U - i} \\ P_V &= \sum_{i=v_V+1}^{n_V} \binom{n_V}{i} p_{max}^i (1 - p_{max})^{n_V - i} \\ P_M &= \sum_{i=v_M+1}^{n_M} \binom{n_M}{i} p_M^i (1 - p_M)^{n_M - i}. \end{aligned} \tag{88}$$

Due to the notation complexity we omit the derivation of Ineq. (87). Instead, below we give an alternate method that achieves the same result.

The key observation is that P_U represents the probability with which more than $\sum_{i \in U} \lfloor u \cdot r_i \rfloor + v_U$ packets from flows in U arrive during the interval $[t_0, t_0 + u)$. This is easy to see as the probability that *exactly* $\sum_{i \in U} \lfloor u \cdot r_i \rfloor + m$ packets from flows in U arrive during $[t_0, t_0 + u)$ is $\binom{n_U}{m} p_{min}^m (1 - p_{min})^{n_U - m}$ (see Eq. (69) for comparison).

Similarly, P_V is the probability that more than $\sum_{i \in V} \lfloor u \cdot r_i \rfloor + v_V$ packets from flows in V arrive during $[t_0, t_0 + u)$, while P_M is the probability that more than $\sum_{i \in M} \lfloor u \cdot r_i \rfloor + v_M$ packets from flows in M arrive during the same interval.

Consequently, $(1 - P_U)(1 - P_V)(1 - P_M)$ represents the probability with which *no* more than $\sum_{i \in U} \lfloor u \cdot r_i \rfloor + v_U$, $\sum_{i \in V} \lfloor u \cdot r_i \rfloor + v_V$, *and* $\sum_{i \in M} \lfloor u \cdot r_i \rfloor + v_M$ packets are received from flows in U, V, and M during $[t_0, t_0 + u)$. Clearly this probability is *no larger* than the probability of receiving no more than $\sum_{i=1}^{n} \lfloor u \cdot r_i \rfloor + v$ packets from *all* flows during the interval $[t_0, t_0 + u)$, probability which is exactly $1 - P^f(x > s, u)$. This yields

$$
\begin{aligned}
1 - P^f(x > s, u) &\geq (1 - P_U)(1 - P_V)(1 - P_M) \Rightarrow \\
P^f(x > s, u) &\leq 1 - (1 - P_U)(1 - P_V)(1 - P_M) \leq P_U + P_V + P_M.
\end{aligned} \tag{89}
$$

Next, consider the expression of P_U in Eq. (88). Let

$$
s_U = v_U - u_U, \tag{90}
$$

where $u_U = p_{min} n_U$. Then it is easy to see that the expressions of p_{min} (i.e., $p_{min} = u_U / n_U$) and P_U, given by Eq. (88), are identical to the expressions of $p(d)$ and $P(x > s, u)$, given by Eqs. (48) and (50), respectively, after the following substitutions: $d \leftarrow u_U$, $n \leftarrow n_U$, $u \leftarrow u_U$, $s \leftarrow s_U$. By applying the result of Lemma 6 we have the following bound

$$
\begin{aligned}
P_U &= \sum_{i = u_U + s_U + 1}^{n_U} \binom{n_U}{i} p_{min}^i (1 - p_{min})^{n_U - i} \\
&< \beta(n_U) \sqrt{\frac{1}{2\pi}} \left(\frac{1 - (s_U - 1)/2n_U}{1 + (s_U - 1)/2n_U} \right)^{2 s_U} \frac{(n_U + s_U)^2}{4 s_U n_U}.
\end{aligned} \tag{91}
$$

Next we compute s_U, such that

$$
\frac{\varepsilon}{3} = \beta(n_U) \sqrt{\frac{1}{2\pi}} \left(\frac{1 - (s_U - 1)/2n_U}{1 + (s_U - 1)/2n_U} \right)^{2 s_U} \frac{(n_U + s_U)^2}{4 s_U n_U}. \tag{92}
$$

By applying the same approximations used in proving Lemma 7 (see Ineq. (63)), i.e., $s_U \ll n_U$, $s_V \ll n_V$, and $s_M \ll n_M$, respectively, we get

$$s_U \simeq \sqrt{n_U \left(\frac{\ln n_U}{2} - \frac{\ln(\varepsilon/3)}{2} - 1 \right)}, \tag{93}$$

and similarly

$$\begin{aligned} s_V &\simeq \sqrt{n_V \left(\frac{\ln n_V}{2} - \frac{\ln(\varepsilon/3)}{2} - 1 \right)} \\ s_M &\simeq \sqrt{n_M \left(\frac{\ln n_M}{2} - \frac{\ln(\varepsilon/3)}{2} - 1 \right)}. \end{aligned} \tag{94}$$

By using the above values for s_U, s_V, and s_M, respectively, and by the definition of $P^f(x > s, u)$ and Ineq. (87), we have

$$P(x > s, u) \leq P^f(x > s, u) \leq P_U + P_V + P_M \leq 3 \cdot \frac{\varepsilon}{3} = \varepsilon. \tag{95}$$

Now it remains to compute s. First, recall that $s_U = v_U - u_U$, $s_V = v_U - u_V$, $s_M = v_M - u_M$, where $u_U = p_{min}u$, $u_V = p_{max}u$, and $u_M = p_M u$ (see Eq. (90)). From here we obtain

$$\begin{aligned} s_U + s_V + s_M &= (v_U - n_U) + (v_V - n_V) + (v_M - n_M) \\ &= v - n_U - n_V - n_M - 3 \\ &= v - p_{min}n_U - p_{max}n_V - p_M n_M \\ &= v - \sum_{i \in U} p_{min} - \sum_{i \in V} p_{max} - \sum_{i \in M} p_M \\ &= v - \sum_{i=1}^{n} p_i = s. \end{aligned} \tag{96}$$

As both $P(x > s, u)$ and $P^f(x > s, u)$ decrease in s, for our purpose it is sufficient to determine an upper bound for s. From Eqs. (93), (94) and (96) this reduces to compute

$$\max \left(\sum_{I \in \{U,V,M\}} \sqrt{n_I \left(\frac{\ln n_I}{2} - \frac{\ln(\varepsilon/3)}{2} - 1 \right)} \right), \tag{97}$$

subject to $n_U + n_V + n_M = n$. Since the function $\sqrt{x \ln x}$ is concave, it follows that expression (97) achieves maximum for $n_U = n_V = n_M = n/3$. Finally, we choose

$$s = 3 \cdot \sqrt{\frac{n}{3} \left(\frac{\ln(n/3)}{2} - \frac{\ln(\varepsilon/3)}{2} - 1 \right)} = \sqrt{3n \left(\frac{\ln n}{2} - \frac{\ln \varepsilon}{2} - 1 \right)}, \tag{98}$$

which completes the proof. □

By combining Lemmas 7 and 10 we have the following result

Theorem 2 *Consider a server traversed by n flows. Assume that the arrival times of the packets from different flows are independent, and that all packets have the same size. Then, for any given probability ε, the queue size at any time instant during a server busy periodic is asymptotically bounded above by s, where*

$$s = \sqrt{\beta n \left(\frac{\ln n}{2} - \frac{\ln \varepsilon}{2} - 1 \right)}, \tag{99}$$

with a probability larger than $1 - \varepsilon$. For identical reservations $\beta = 1$; for heterogeneous reservations $\beta = 3$.

Appendix D: Proof of Theorem 3

Theorem 3 *Consider a link of capacity C at time t. Assume that no reservation terminates and there are no reservation failures or request losses after time t. Then if there is sufficient demand after t the link utilization approaches asymptotically $C(1-f)/(1+f)$.*

Proof. If the aggregate reservation at time t is larger than $C(1-f)/(1+f)$, the proof is trivially true. Next, we consider the case in which the aggregate reservation is less than $C(1-f)/(1+f)$.

In particular, let $C(1-f)/(1+f) - \Delta$ be the aggregate reservation at time t. Without loss of generality assume $t = u_k$. Then we will show that if no reservation terminates, no reservation request fails, and it is enough demand after time u_k, then at least $(1+f)\Delta/2$ bandwidth is allocated during the next two slots. i.e., during the interval $(u_k, u_{k+2}]$. Thus, for any arbitrary small real ε, we are guaranteed that after at most

$$2 \times \frac{\ln(\varepsilon/\Delta)}{\ln((1-f)/2)} \tag{100}$$

slots the aggregate reservation will exceed $C(1-f)/(1+f) - \varepsilon$.

From Eq. (20) it follows that the maximum capacity which can be allocate during the interval $(u_k, u_{k+1}]$ is $\max(C - R_{cal}(u_k), 0)$. Assume then that Δ_1 capacity is allocated during $(u_k, u_{k+1}]$, where $\Delta_1 \leq \max(C - R_{cal}(u_k), 0)$. Consider two cases whether $\Delta_1 \geq \Delta$ or not. If $\Delta_1 \geq \Delta$ the proof follows trivially.

Assume $\Delta_1 < \Delta$. Then we will show that at time u_{k+2} the aggregate reservation can increase by at least a constant fraction of Δ. From Figure 16 is easy to see that for any reservation continuously active during an interval $(u_k, u_{k+1}]$ we have

$$b_i(u_k, u_{k+1}) < r_i(T_W + T_I + T_J). \tag{101}$$

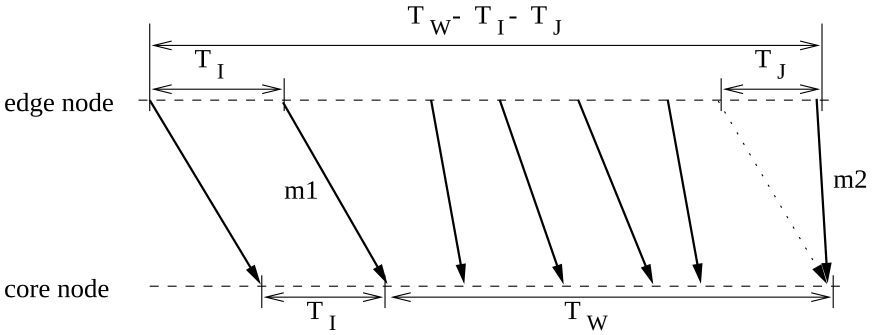

Figure 16: The scenario in which the upper bound of b_i, i.e., $r_i(T_W - T_I - T_J)$, is achieved. The arrows represent packet transmissions. T_W is the averaging window size; T_I is an upper bound on the packet inter-departure time; T_J is an upper bound on the delay jitter. Both $m1$ and $m2$ fall just inside the estimation interval, T_W, at the core node.

Since no reservation terminates during $(u_k, u_{k+1}]$ we have $\mathcal{L}(u_{k+1}) = \mathcal{L}(u_k) \cup \mathcal{N}(u_{k+1})$. Let $ac_i \in (u_k, u_{k+1}]$ be the time when flow i becomes active during $(u_k, u_{k+1}]$. Since $b_i(ac_i, u_{k+1}) \leq b_i(u_k, u_{k+1})$, by using Eq. (101) we obtain

$$B(u_k, u_{k+1}) = \sum_{i \in \mathcal{L}(u_{k+1})} b_i(u_k, u_{k+1}) < \sum_{i \in \mathcal{L}(u_{k+1})} r_i(T_W + T_I + T_W). \tag{102}$$

From here we get

$$R_{DPS}(u_k, u_{k+1}) \quad < \quad R(u_{k+1})(1+f). \tag{103}$$

Since there are no duplicate requests or partial reservation failures after time $t = u_k$, we have $\Delta_1 = R_{new}(u_{k+1})$. From here and from Eq. (20) and Ineq. (103) we have

$$R_{cal}(u_{k+1}) \leq \frac{R_{DPS}(u_{k+1})}{1-f} + \Delta_1 < R(u_{k+1})\frac{1+f}{1-f} + \Delta_1. \tag{104}$$

In addition, we have $R(u_{k+1}) = R(u_k) + \Delta_1$. Since $R(u_k) = C(1-f)/(1+f) - \Delta$, from Eq. (104) it follows

$$C - R_{cal}(u_{k+1}) \geq C - R(u_{k+1})\frac{1+f}{1-f} - \Delta_1 \geq \frac{1+f}{1-f}\Delta - \frac{2}{1-f}\Delta_1. \tag{105}$$

Finally, consider two cases whether (a) $\Delta_1 < \Delta(1+f)/2$, or (b) not. If (a) is true then the link can allocate up to

$$\Delta_1 + C - R_{cal}(u_{k+1}) > \Delta_1 + \frac{1+f}{1-f}\Delta - \frac{2}{1-f}\Delta_1 = \frac{1+f}{1-f}(\Delta - \Delta_1) > \frac{1+f}{2}\Delta, \tag{106}$$

capacity during the time interval $(u_k, u_{k+2}]$. In case (b) we have trivially $\Delta_1 \geq \Delta(1+f)/2$. Thus in both cases we can allocate at least $\Delta(1+f)/2$ new capacity during $(u_k, u_{k+2}]$. $\Box$